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IFRS 9 Measurement of Financial Instruments 2018: Jameel's Non-Normal Brownian Motion Models are Indeed IFRS 9 Complaint Models

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Abstract

The measurement of Financial Instruments under IFRS 9 requires the incorporation of forward-looking information and Economic forecasts of the future macroeconomic scenarios into the existing Accounting, Banking and Economic Models. In this paper, the author considered Geometric Brownian Motion, Biagin, Cox-Ingersoll-Ross, Ornstein-Uhlenbeckprocess, Vasicek, Black-Karasinki, Chen, Kalotay-Williams-Fabozzi, Longstaff-Schwatz, Ho-Lee, Hull and White, and Black-Derman-Toy Models for Pricing Stocks, Bitcoin, Indexes, ETFs, and Leveraged ETFs, Bonds, Interest Rate Movements, Caps, Floors, European Swaptions, and Bond Options thereby incorporating forward-looking information $\{W_{JB(t)}\}$ satisfying Jameel's Criterion and Geometric average of only positive Economic forecasts of the future Macroeconomic parameters $\{(\mu_A)\ \text{and}(\sigma_A)\}$ using Jameel's Contractional-Expansional Stress Methods and Jameel's substitutions $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, μ_A is POSITIVE INFINITESIMAL, $\sigma_A \geq 1$ and define σ_A as Geometric Volatility of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and μ_A as Geometric Means ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters. The paper replaces the Wiener Process $\{W(t): t \geq 0\}$ in the existing models with the following proposed NON-NORMAL STRESS CONDITIONS: (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if$\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if$\sigma_A = 1$, $\mu_A = 0$. The paper tested the performances of only proposed STOCKS stressed closed form models using Chevron Corporation (CVX) Stock data extracted from yahoo finance, time series from 2014 – 1991. The results were fascinatingly interesting, impressive, viable and reliable, sophisticated, and complaint with IFRS 9 since they incorporated forward-looking information and Economic forecasts of the future macroeconomic parameters thereby minimizing the differences between market prices and models prices.

Keywords: Stocks, Bonds, ETFs, Bitcoin, Derivatives, Jameel's Stressed Closed Form Prices

1. Introduction

The IASB in July, 2014 issued the final version of IFRS 9 Measurement of Financial Instruments beginning on or after 1st January, 2018 with early adoption permitted. It replaces IAS 39 Financial Instruments: Recognition and Measurement. The major target of accounting standards is to provide financial information that stake-holders would find useful when making decisions. The most challenging aspects required by IFRS 9 are the treatment on incorporation of forward-looking information and economic forecasts of future macroeconomic scenarios into the existing Accounting, Banking and Economic Models.

A forward- looking calculations should be based on accurate estimation of current and future financial instruments prices. Barnaby Black, Shirish Chinchalkar, Juan M. Licari (2016) argued that building and implementing Econometric Models for different asset classes, the modeler needs to carefully examine the requirements from the perspective of final users of the models. Also, they stated that Regulatory Stress Testing requires that the models should demonstrate sensitivity to macroeconomic conditions. According to Evert de Vries and Martijinde Groot (2016), the forward-looking Economic Forecasts of the credible and robust future macroeconomic scenarios are commonly at the domain of economic research Departments. Macroeconomic forecasting concentrates mainly on Country-specific variables. Growth of Domestic Product, Unemployment Rates, Inflation Indices and Interest Rates are typically projected variables. Usually, only large International Banks with an economic research Department are able to project consistent economic outlooks and scenarios. More so, with advancement in Economic and Financial Software developments, nowadays, there exist sophisticated macroeconomic forecasting softwares available that could be used to predict fundamental macroeconomic parameters.

The IFRS 9 accounting rules regarding Measurement of Financial Instruments will NORROW the wide gaps between Financial Instruments Market Prices and Models Prices as Jameel's Advanced Stressed Models do. The major reason of IFRS 9 was that *"The Credit Risk at origination is included in the pricing of Financial Asset but any increase in Credit Risk is NOT"*.

Jamilu (2015) has attempted to incorporate increase in Credit Risk in the existing Expected Credit Loss Model, Derivatives and Assets Pricing Models using Jameel's Criterion and to come up with Jameel's Advanced Stressed Models. Jameel's Criterion and Jameel's Advanced Stressed Models were first introduced to financial market in July, 2015. Jameel's Criterion (2015) is a set of axioms provided for underlying assets return probability distributions to satisfy in order to be incorporated in the Jameel's Advanced Stressed Models to enable them capture Low-Probability, High-Impact Events, making the existing predictive models more sophisticated, robust, reliable and to traces the trajectories of the current and future economic and financial crises. Jameel's Advanced Stressed Models (2015) are advanced models stressed to capture Low-Probability, High-Impact Events, making the existing predictive models sophisticated, robust, and reliable such that they can traces the trajectories of the current and future economic and financial crises using underlying assets return probability distributions that SATISFIED Jameel's Criterion.

In this paper, the Author attempted to INCORPORATE forward-looking information $\{W_{JB(t)}\}$ satisfies Jameel's Criterion and Geometric average of only positive Economic forecasts of the future Macroeconomic scenarios $\{(\mu_A)\text{ and}(\sigma_A)\}$ using Jameel's Contractional-Expansional Stress Methods and Jameel's substitutions $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, $\mu_A$is POSITIVE INFINITESIMAL, $\sigma_A \geq 1$ and define σ_A as Geometric Volatility of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and μ_A as Geometric Means of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters. Note that $\{W_{JB(t)}\}$ is a Non-Normal Brownian Motion variable fat-tail stochastic probability distribution of the considered Financial Instrument Return Satisfies Jameel's Criterion. The paper replaces the Wiener Process $\{W(t): t \geq 0\}$ in the existing models with the following proposed NON-NORMAL STRESS CONDITIONS: (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if$\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if$\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if$\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if$\sigma_A = 1$, $\mu_A = 0$. Finally, the paper round up with the test of performances of only proposed STOCKS stressed closed form models using Chevron Corporation (CVX) Stock data extracted from yahoo finance, time series from 2014 – 1991.

2. Materials and Methods

2.1 Materials

2.1.1 Stochastic Process

A stochastic (uncertainty) process can be defined as a Mathematical object usually described as a collection of random variables or can be defined as numerical values of some system randomly changing over time, for instance movement of a gas molecule.

2.1.2 Random Walk

A cornerstone of the theory of stochastic processes is called a Random Walk

2.1.3 General Stochastic Integral

The General Stochastic Integral is given by: $\int_0^t X(s)dM(s), t \geq 0$, where $X \equiv \{X(t): t \geq 0\}$ and $M \equiv \{M(t): t \geq 0\}$ are both *Stochastic Process.*

2.1.4 Normal Distribution

(a) The density of the normal distribution is expresses in the following way:

$\varphi(x) = \frac{1}{\sqrt{2\sigma^2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right); \sigma > 0, \mu \in R$, if $\mu = 0$ and $\sigma = 1$, one calls this density of the standard normal distribution.

(b) The distribution function of the standard normal distribution is expressed in the following manner: $\Phi(z) = P(\{X \leq z\}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-x^2/2)dx$

(c) If the random variable X is normally distributed with parameters $\mu \in R$ and $\sigma > 0$, one write $X \sim N(\mu, \sigma^2)$. A normally distributed random variable can adopt the values from the entire R and it is true that: Expected Values: $E[X] = \mu$ and the Variance : $Var[X] = \sigma^2$.

2.1.5 Brownian Movement

The stochastic process $\{W(t), t \in [0, T]\}$ which is designated as the Brownian movement or also the Wiener Process. It is determined by the fact that $W(t)$ is NORMALLY DISTRIBUTED random variable with expected value zero and variance t, therefore it is true that: $W(t) \sim N(0, t)$. The Geometric Brownian Motion (GBM) assumes that stock prices are Log-Normally Distributed with a mean of the certain component and a standard deviation of the uncertain component that $\ln \frac{S_T}{S_0} \sim \Phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\varepsilon\sqrt{T}\right]$.

S_0 : Initial Stock Price; μ : Expected Annual Return; σ : Expected Annual Volatility

2.1.6 Fractional Brownian Motion

Let $H \in (0,1)$ then Stochastic process $B^H : [0, +\infty] \times \Omega \to R$ which is GAUSSIAN, H-self-similar has stationary increments and $\sigma^2 = 1$ is called the Fractional Brownian Motion. Mandelbrot and Van Ness (1968) suggested the use of Fractional Brownian Motion Model with adaptive parameters as a source of randomness for the

FINANCIAL MARKET. The Fractional Brownian Motion process $\{B_t^H, t \geq 0\}$ with Hurst Index H is a Centered GAUSSIAN process. If $H = 0.5$, then $\{B_t^H, t \geq 0\}$ is a standard Brownian Motion process. $H \neq 0.5$ then $\{B_t^H, t \geq 0\}$ is neither a semimartingle nor a Markov process. For $H \neq 0.5$ case, the $\{B_t^H, t \geq 0\}$ is represented by Mandelbrot and Van Ness: $B_t^H = \frac{1}{\Gamma(1+\alpha)}[Z_t + B_t]$, $H \in (0,1)$. Recall that $dS_t = S_t(\mu dt + \sigma dW_t)$ then the driving process W_t is replaced by a Fractional Brownian Motion process B_t^H with adaptive parameters μ and σ. In this case, the model can be represented by the stochastic differential equation (SDE) as shown: $dS_t = S_t(\mu.dt + \sigma.dB_t^H)$, μ and σ are adaptive parameters, the same as the previous model. The $\{B_t^H, t \geq 0\}$ is a Fractional Brownian Motion Process, hence is called a Fractional Brownian Motion Model with adaptive parameters (FBMAP). When $H = 0.5$ it becomes a BH. Thus FBMAP becomes BM.

2.1.7 Ito's Lemma

Let start with *Stochastic Process* satisfying a *Stochastic Differential Equation (SDE)* then we proceed to **Ito's Lemma.**

Suppose that X is a Stochastic Process satisfying the ***Stochastic Differential Equation (SDE)*** given by: $dX = adt + bdB$, where B is Brownian Motion, by which we mean $dX(t) = a(X(t),t)dt + b(X(t),t)dB(t)$, where a and b are real-valued functions on R^2, by which we mean the X satisfies the integral equation: $X(t) = \int_0^t a(X(s),s)ds + \int_0^t b(X(s),s)dB(s)$, where the last integral is defined as an ***Ito Integral***. Such a process X is often called as ***Ito Process.*** Note that the process X appears on both sides of the above equation, but the value at t given on the left depends only on the values at times s and $s \leq t$. Assuming that X has continuous paths, it suffices to know $X(s)$ for all $s < t$ on the right. Nevertheless, there is a need for supporting theory (which has been developed) about the existence and uniqueness of solution to the integral (or equivalently the SDE).

An elementary example arises when $X(t) = \mu t + \sigma B(t)$, where μ and σ are constants. Then from the above equation $a(x,t) = \mu$ and $b(x,t) = \sigma$, independent of x and t. Then we can directly integrate the ***SDE*** to see that the process is BM with drift μ and ***diffusion coefficient*** σ^2. Another important example is standard Geometric Brownian Motion (GBM) for which $a(x,t) = \mu x$ and $b(x,t) = \sigma x$. Letting the stock price at time t be $S(t)$, we write the classical GBM SDE as: $dS = Sdt + \sigma SdB$, where again μ and σ are constants. Note that S appears in both terms on the right.

Now, assume given the Ito process X and suppose that $f: \mathbb{R}^2 \to \mathbb{R}$ is a smooth function, with continuous second derivatives. ***Ito's lemma concludes*** that $Y(t) \equiv f(X(t),t)$ has an SDE representation with

$$dY = \left(f_t + af_x + \frac{1}{2}b^2 f_{xx}\right)dt + bf_x dB.$$

Example 1

Suppose that we now consider the logarithm: $\ln(S(t)/S(0)) = \ln(S(t)) - \ln(S(0))$. We can apply the function $f(x,t) = \ln(x)$, for which $f_x = 1/x$, $f_{xx} = -1/x^2$ and $f_t = 0$ then we have: $d\ln(S(t)/S(0)) = (\mu - \frac{\sigma^2}{2})dt + \sigma dB$.Thus, $\ln(S(t)/S(0)) = (\mu - \frac{\sigma^2}{2})t + \sigma B(t), t \geq 0$. Note that the drift of this Brownian Motion is not μ. The drift terms in the two specifications do not agree. Given that $\ln(S(t)/S(0)) = vt + \sigma B(t), t \geq 0$, we get $E[S(t)] = S(0)\exp(v + \sigma^2/2)t, t \geq 0$,whereas from the SDE, it would be $E[S(t)] = S(0)\exp(\mu t)$. The parameters $\mu and v$ in these two representations should be related by:

$\mu = v + \frac{\sigma^2}{2}$ or $v = \mu - \frac{\sigma^2}{2}$.

In this research paper, the ***Multi-billion Questions reference to Ito's Lemma*** are:

(a) What is $\int_0^T \sigma_t dW_t$

(b) The transformation of***Ito Process***in ***Calculus*** given by: $X_t - X_0 = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$ into ***Stochastic Differential Equation (SDE)***

In view of the above detail explanation, **the Multi-billion answers are: Ito Process** in **Calculus** given by: $X_t - X_0 = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$ can be transformed into **Stochastic Differential Equation (SDE)** as $dX_t = \mu_t dt + \sigma_t dW_t$. Thus, answered question (a).

2.1.8 Jameel's Criterion

Under this criterion, we run the goodness of fits test such that:

i. We accept if the Average of the ranks of Kolmogorov Smirnor,Anderson Darling and Chi-squared is less than or equal to Three (3)

ii. We must choose the Probability Distribution follows by the data **ITSELF** regardless of its Rankings

iii. If there is tie, we include both the Probability Distributions in the selection

iv. At least Two (2) Probability Distributions must be included in the selection

v. We select the most occur Probability Distribution as the qualify candidate in each case of test of goodness of fit.

vi. **Criterion Enhancement Axiom:**Thode (2012) intensively discussed about the Best Goodness of Fit Tests such as Kolmogorov Smirnov (KS) Test, Anderson-Darling Test, Jarque and Bera (JB) Test, Shapiro Wilk (SW) Test, Cramer-Von Mises Test, Pearson $(\chi^2 \ Godness \ of \ Fit)$ Test, Lilliefors Corrected K-S Test, D'AgostinoSkewness Test, Anscombe-Glynn Kurtosis Test, D'Agostino-Pearson Omnibus Test. Let $\{T_1, T_2, ..., T_n\}$ be the set of such Best Goodness of Fit Tests, $\{x_1, x_2, ..., x_n\}$ be their **RANKS** respectively then the generality of (i) can be expressed (or enhanced) if $\frac{(x_1 + x_2 + ... + x_n)}{n} \leq a$, where $0 < a \leq n, \ n \in N$ or equivalently, $x_1 + x_2 + ... + x_n \leq an$.

vii. **Last Unit Axiom:** let $W_{JB}(t)$ be such that it satisfied axioms (i) to (iv). Let $\{r_1, r_2, ..., r_n\}$ be the ranks of fitness test of $W_{JB}(t)$ obtained from the tests $\{T_1, T_2, ..., T_n\}$ respectively then if $\forall i \in \{1, 2, ..., n\}, \ r_i = 1$ regardless of the Time Series, Company and so on. Consequently, if for all fitness test runs, turn out to be the same $W_{JB}(t)$ then the **PREDICTED PRICE PATH** will finitely coincides many times with the **REAL PRICE PATH** of the stock under consideration.

2.1.9 Top Fat-Tailed Probability Functions using Jameel's Criterion as of 2015

Using Jameel's Criterion, Jamilu (2015) considered Eleven (11) out of Fifty (50) World's Biggest Public Companies by FORBES as of 2015 Ranking regardless of the platform in which they are listed, Number of the Research Companies, Time Series (Short or Long), Old or Recently listed Companies using the time series from 2014 – 2009 with the aim of finding the Best Fitted Fat – Tailed Stocks Probability Distributions. However, in this research paper, the Author considered Top Two (2) and 4th Stocks Fat-Tailed Probability Functions thereby comparing the performances of the *Proposed Jameel's Stressed Closed Form Prices, Normal (Standard Brownian Motion) Prices with Market (Real) Prices* as shown below:

Log – Logistic (3P) Probability Distribution (1st):

$$f(x;\mu,\sigma,\xi)=\frac{\left(1+\frac{\xi(x-\mu)}{\sigma}\right)^{-\left(1/\xi+1\right)}}{\left[1+\left(1+\frac{\xi(x-\mu)^{-1/\xi}}{\sigma}\right)\right]^{2}}\quad;\quad x\geq\mu$$

Cauchy Probability Distribution (2nd):

$$f(x;\mu,\sigma,\pi)=\left(\pi\,\sigma\left(1+\left(\frac{x-\mu}{\sigma}\right)^{2}\right)\right)^{-1}\quad;\quad -\infty<x<+\infty$$

Burr (4P) Probability Distribution (4th):

$$f(x;\alpha,\beta,\gamma)=\frac{ak\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}};\quad \alpha,\beta,k>0$$

2.2 Methods

2.2.1 Stocks Pricing for IFRS 9 Compliance

Let define Brownian movement (Motion). If some quantities are constantly undergoing small, random fluctuations then we say it is undergoing a Brownian motion or in Physics can be defined as a random movement of particles in a fluid due to their collision with other atoms or molecules. This can be expressed as:

$$Brownian\ Motion:=\begin{Bmatrix}Expected Component\\ or\\ Certain\ Component\end{Bmatrix}+\begin{Bmatrix}Un\exp ected\,Component(Stochastic\ Term)\\ or\\ Uncertain\,Component(Rondom Component)\end{Bmatrix}$$

Mathematically, $S(t) = \mu t + \sigma W(t),\ \mu \in R,\ \sigma \geq 0$, where μ is the drift or annual expected change; σ is the annual volatility and $W(t)$ a Wiener Process. If $\ln S(t)$ follows Brownian Motion then $\ln S(t)$ can be expressed as $\mu_0 t + \sigma_0 W(t)$.

Recall that $R_t = \frac{S_t - S_{t-k}}{S_{t-k}} \Rightarrow 1 + R_t = \frac{S_t}{S_{t-k}}$ where, $r_t = \ln(1 + R_t) \Rightarrow r_t = \ln\left(\frac{S_t}{S_{t-k}}\right) \Rightarrow \ln S_t - \ln S_{t-k}$

$$r_t = \ln S_t - \ln S_{t-k} = \ln\left(\frac{S_t}{S_{t-k}}\right) = \frac{S_t - S_{t-k}}{S_{t-k}}.$$

$$\text{Thus, } \ln\left(\frac{S_t}{S_{t-k}}\right) = \ln S_t - \ln S_{t-k} = \frac{S_t - S_{t-k}}{S_t} = \frac{dS(t)}{S(t)} = \mu t + \sigma dW(t)$$

Hence, $\frac{dS(t)}{S(t)} = \mu t + \sigma dW(t)$. Now, suppose that to each point of a sample, we assign a number. We then have a Function defined on the sample space. This function is called a random variable (or stochastic variable) or precisely a random function (stochastic function) given by $P(X = x_k) = f(x_k); k = 1,2,...$ or simply $P(X = x) = f(x)$ is a probability function (probability distribution) or Random Function (Stochastic Function).

The Brownian Motion $S(t) = \mu t + \sigma W(t),\ \mu \in R,\ \sigma \geq 0$, the Wiener Process $W(t)$ is indeed a Random GAUSSIAN (NORMAL) Function with mean zero and variance t as shown by Norbert-Wiener in the early 1920s. Mathematically, $W(t)$ is a NORMALLY DISTRIBUTED random variable with expected value zero and variance t. Therefore it is true that $W(t) \sim N(0, t)$. From the fact that the process $\{W(t) : t \geq 0\}$ is a GAUSSIAN (NORMAL) with mean zero and variance t then $S(t) = \mu t + \sigma W(t)$ is a NORMAL BROWNIAN MOTION STOCK PRICE.

Now, as argued by (Kou (2002); Abidin and Jaffar (2014); Marathe and Ryan (2005); Gajda and Wylomanka (2012)) that the NORMAL and FRACTIONAL Brownian Motion as well as the Stable Distributions have the following WEAKNESSES:

(a) Difficulties in identifying the right tail distribution (process) whether to use power-type or exponential-type distributions;

(b) The stable distributions generalize normal distribution;

(c) Geometric Brownian Motion (GBM) can only be used to forecast maximum of two weeks closing prices;

(d) Geometric Brownian Motion (GBM) does not include cyclical or seasonal effects; and

(e) Geometric Brownian Motion (GBM) does not account for periods of constant values, they observed periods where prices stay on the same level, particularly true for asset with low liquidity.

Also, Levy processes provide a natural generalization of the sum of independent and identically distributed (iid) random variables. The simplest possible levy processes are the standard Brownian motion $W(t)$, Poisson processes $N(t)$, and compound Poisson processes $\sum_{i=1}^{N(t)} Y_i$, where Y_i are (iid) random variables. It is not clear how heavy the tail distributions although, as some people favor power-type distributions other exponential-type distribution, although as pointed out by Kou (2002, P.1090), the power-type right tails cannot be use in models with continuous compounding as they lead to infinite expectation for the asset price. In view of the foregoing, we PROPOSE the following LEMMA:

2.2.2 Proposed Jameel's Lemma

(a) Prediction of Future Stock Prices: Let $\{W(t): t \geq 0\}$ be a Gaussian (Normal) with mean zero and variance t. Let $S(t)$ be a Stock Price given by the Stochastic process $S(t) = \mu t + \sigma W(t); \mu \in R, \sigma \geq 0$. Let $\{W_{JB}(t): t \geq 0\}$ be a Fat-Tail Stochastic or Random Probability Function satisfied JAMEEL'S CRITERION then the NON-NORMAL BROWNIAN MOTION STOCK PRICE can be expressed as:

$$S_{JB}(t) = \mu.t + \sigma.W_{JB}(t)$$

$\mu \in R, \sigma \geq 0$ are adaptive parameters and the same as in the Normal Model above.

If $\frac{dS(t)}{S(t)} = \mu t + \sigma dW(t)$ follows NORMAL BROWNIAN MOTION then

$\frac{dS_{JB}(t)}{S_{JB}(t)} = \mu t + \sigma dW_{JB}(t)$ will FOLLOW NON-NORMAL BROWNIAN MOTION where, $W_{JB}(t)$ satisfied Jameel's Criterion. Then integrating the NON-NORMAL BROWNIAN MOTION equation, we have:

$$\int \frac{dS_{JB}(t)}{S_{JB}(t)} = \int \mu dt + \int \sigma dW_{JB}(t) \Rightarrow \ln(S_{JB}(t)) = \mu t + \sigma W_{JB}(t) + c$$

$S_{JB}(t) = e^c . e^{(\mu t + \sigma W_{JB}(t))}$. Let $p_0 = e^c$ then

$$S_{JB}(t) = p_0 . \exp(\mu . t + \sigma . W_{JB}(t))$$

Alternatively, $dS_{JB}(t) = S_{JB}(t)[\mu dt + dW_{JB}(t)]$ then in order to solve for $S_{JB}(t)$ we apply **Ito**to $d \ln S_{JB}(t)$:

$$d \ln S_{JB}(t) = \frac{1}{S_{JB}(t)} dS_{JB}(t) - \frac{1}{2} \frac{1}{S_{JB}(t)^2} dS_{JB}(t)^2$$

$$= \frac{1}{S(t)} S_{JB}(t)\left[\mu dt + \sigma dW_{JB}(t)\right] - \frac{1}{2}\frac{1}{S_{JB}(t)^2} S_{JB}(t)^2 \left[\sigma^2 dW_{JB}(t)^2\right]$$

$$d \ln S_{JB}(t) = \mu dt + \sigma dW_{JB}(t) - \frac{1}{2}\sigma^2 dt$$

Then we integrate and apply the fundamental theorem of calculus to get:

$$\ln S_{JB}(t) - \ln S(0) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_{JB}(t)$$

Thus, $S_{JB}(t) = S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_{JB}(t)\right)$.

And the EXPECTATION VALUE is given by:

$$E[S_{JB}(t)] = E\left[S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_{JB}(t)\right)\right]$$

$E[S_{JB}(t)] = S(0)\exp\left(\mu - \frac{1}{2}\sigma^2\right)E[\exp(\sigma W_{JB}(t))]$ depending on $W_{JB}(t)$; $W_{JB}(t)$ satisfied Jameel's Criterion.

This is called NON-NORMAL BROWNIAN STOCK PRICE and can be used to price stocks for the Non-normal times or even at the Normal times since the Normal Brownian Motion Stock model overestimate, where $W_{JB}(t)$ is a Fat-tail Probability Distribution Satisfied Jameel's Criterion.

2.2.3 Propose Jameel's Stressed Closed Form Stock Pricing Models for IFRS 9 Compliance

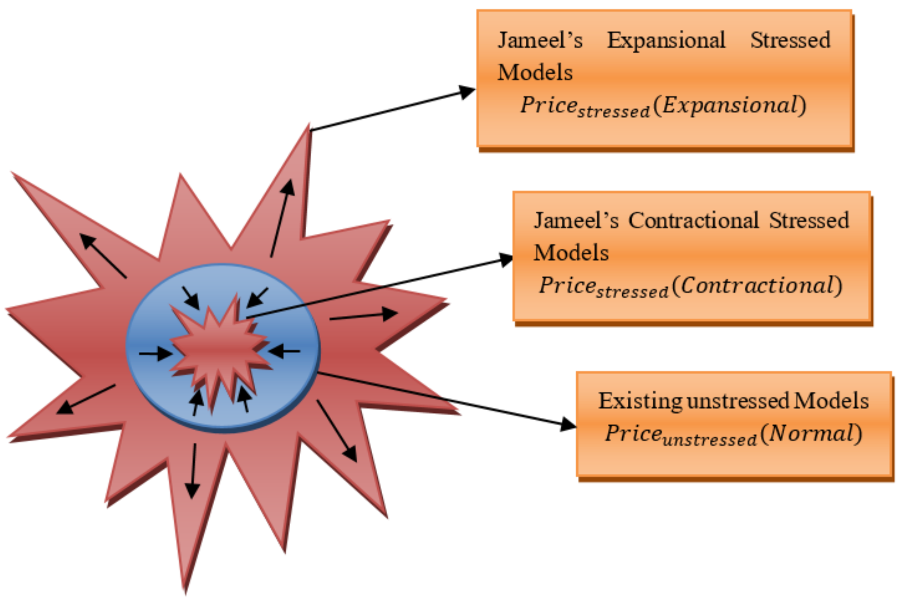

Figure 1. Jameel's Contractional-Expansional Stressed Methods

Jamilu (2017) provided the **CLOSED FORM SOLUTION** of **STOCK PRICE** as: $S_{JB}(t) = p_0\exp(\mu t + \sigma W_{JB}(t)$. Furthermore, in this paper, the Author attempted to come up with other substitutions entitled ***"Jameel's Substitutions for IFRS 9 Compliance"***thereby further **STRESSING THE CLOSED FORM SOLUTION** obtained from the **PROPOSED JAMEEL'S LEMMA** above, then apply Jameel's Criterion and Jameel's Contractional-Expansional Stressed Methods to **REPLACE**$\{W_{JB}\}_{t\geq 0}$ with the **JAMEEL'S SUBSTITUTIONS**

FOR IFRS 9 COMPLIANCE as : (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, $\mu_A = 0$. Define σ_A as Geometric Volatility ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and μ_A as Geometric Means ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters, $\{W_{JB(t)}\}$ is a Non-Normal Brownian Motion variable fat-tail stochastic probability distribution of the considered Financial Instrument Return Satisfying Jameel's Criterion then we have the following ***Propose Jameel's Stressed Closed Form Stocks Pricing Models TYPESfor IFRS 9 Compliance as:***

TYPE 1:

$\left(S_{JB}(t)\right)_{Stressed} = p_0 \exp\left(\mu t + \sigma\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right)$,whenever $\sigma_A > 1$, μ_A is positive infinitesimal;

TYPE 2:

$\left(S_{JB}(t)\right)_{Stressed} = p_0 \exp\left(\mu t + \sigma\left(\mu_A \pm W_{JB}(t)\right)\right)$,whenever $\sigma_A = 1$, μ_A is positive infinitesimal;

TYPE 3:

$\left(S_{JB}(t)\right)_{Stressed} = p_0 \exp\left(\mu t + \sigma\left(\pm\sigma_A W_{JB}(t)\right)\right)$,whenever $\sigma_A > 1$, $\mu_A = 0$;

TYPE 4:

$\left(S_{JB}(t)\right)_{Stressed} = p_0 \exp\left(\mu t + \sigma\left(\pm W_{JB}(t)\right)\right)$,whenever$\sigma_A = 1$, $\mu_A = 0$;

2.2.4 Indexes Pricing for IFRS 9 Compliance

(b) **Prediction of Future Market Indexes:** as in (a) above, the result can be extended to predict future market indexes prices for instance S&P500 (composed of 118 companies from NASDAQ and 382 companies from NYSE), Wilshire 5000, NASDAQ composite Index, Russell 2000, Bottom Line, Nikkei 225, FTSE 100, S&P 100, Dow Jones Industrial Average. Let I be an Index Price of a Market Index M then $\frac{dI}{I} = \mu t + \sigma dW(t),\ \mu \in R,\ \sigma \geq 0$, where $\{W(t): t \geq 0\}$ is a Wiener Process. This is called a Normal Brownian Motion Index Price. For NON-NORMAL INDEX PRICE we have $\frac{dI_{JB}}{I_{JB}} = \mu t + \sigma dW_{JB}(t),\ \mu \in R,\ \sigma \geq 0$ are adaptive parameters and the same as in the Normal case, where $\{W_{JB}(t): t \geq 0\}$ satisfied Jameel's Criterion.

Thus, $I_{JB} = I_0 . \exp(\mu t + \sigma W_{JB}(t))$.

2.2.5 Propose Jameel's Stressed Closed Form Indexes Pricing Models for IFRS 9 Compliance

Applying **JAMEEL'S SUBSTITUTIONS FOR IFRS 9 COMPLIANCE** as in the case of Stocks above, we can generate the following Propose Jameel's Stressed Closed Form Indexes Pricing Models TYPES for IFRS 9 Compliance as:

TYPE 1:

$$\left(I_{JB}(t)\right)_{Stressed} = I_0\exp\left(\mu t + \sigma\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right)$$,whenever $\sigma_A > 1,\ \mu_A$ is positive infinitesimal;

TYPE 2:

$$\left(I_{JB}(t)\right)_{Stressed} = I_0\exp\left(\mu t + \sigma\left(\mu_A \pm W_{JB}(t)\right)\right)$$,whenever $\sigma_A = 1,\ \mu_A$ is positive infinitesimal;

TYPE 3:

$$\left(I_{JB}(t)\right)_{Stressed} = I_0\exp\left(\mu t + \sigma\left(\pm\sigma_A W_{JB}(t)\right)\right)$$,whenever $\sigma_A > 1,\ \mu_A = 0$;

TYPE 4:

$$\left(I_{JB}(t)\right)_{Stressed} = I_0\exp\left(\mu t + \sigma\left(\pm W_{JB}(t)\right)\right)$$,whenever$\sigma_A = 1,\ \mu_A = 0$;

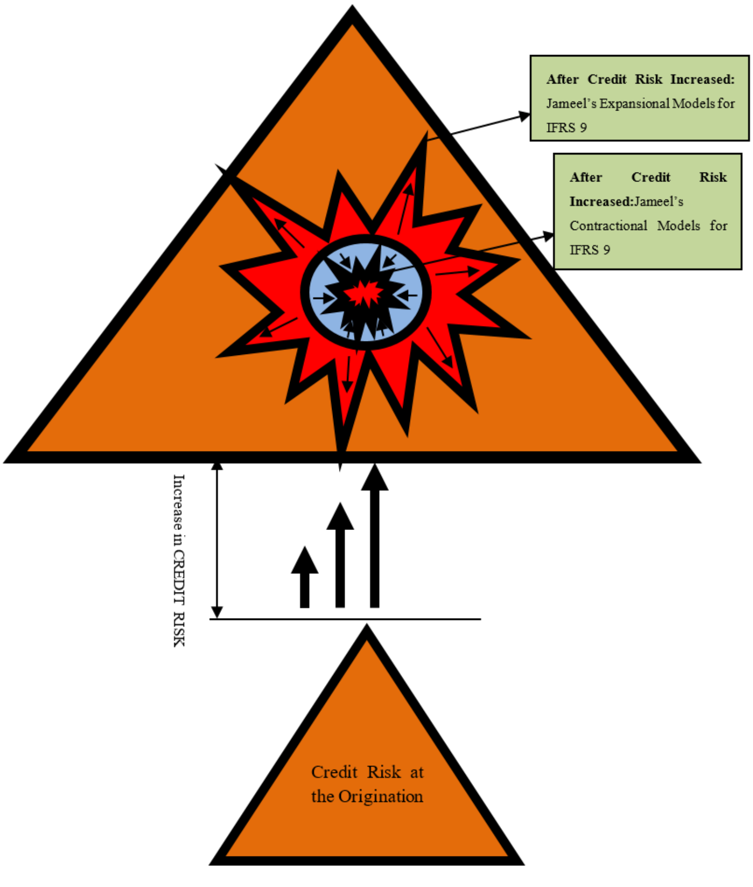

Figure 2. Jameel's Transformational Diagram for IFRS 9 Compliance

Note that the DRIFT μ can be determined for instance assume the Annual Drift (expected Stock Return) = 10%, Annual Volatility = 40%, Initial Stock Price = $100 then Daily Drift = 10/252 = 0.4% trading days per year, Daily Volatility = 40%/sqr(252) = 2.52% because of square root Rule.

Thus, Drift (Mean) = 0.4% - 0.5*(2.5)^2.

Therefore,

$$Drift\ (Mean) = \mu_{daily}(Daily\ Mean\ of\ preceding\ ONE\ YEAR) - \frac{1}{2}\sigma^2_{daily}(ONE\ YEAR)$$

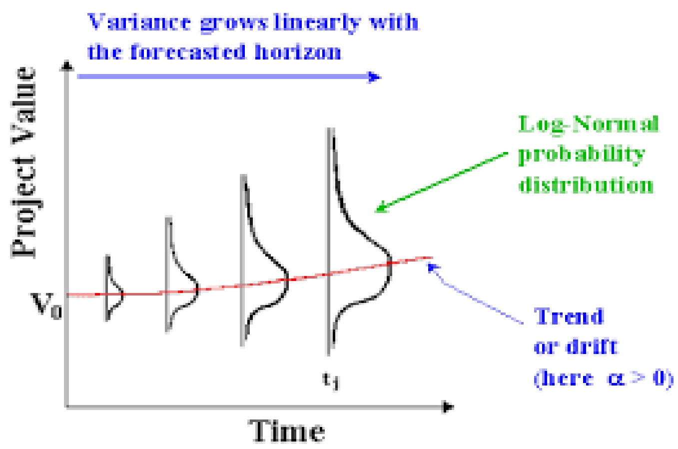

Figure 3. Normal Stocks Brownian Motion

Source: Google Images (2017)

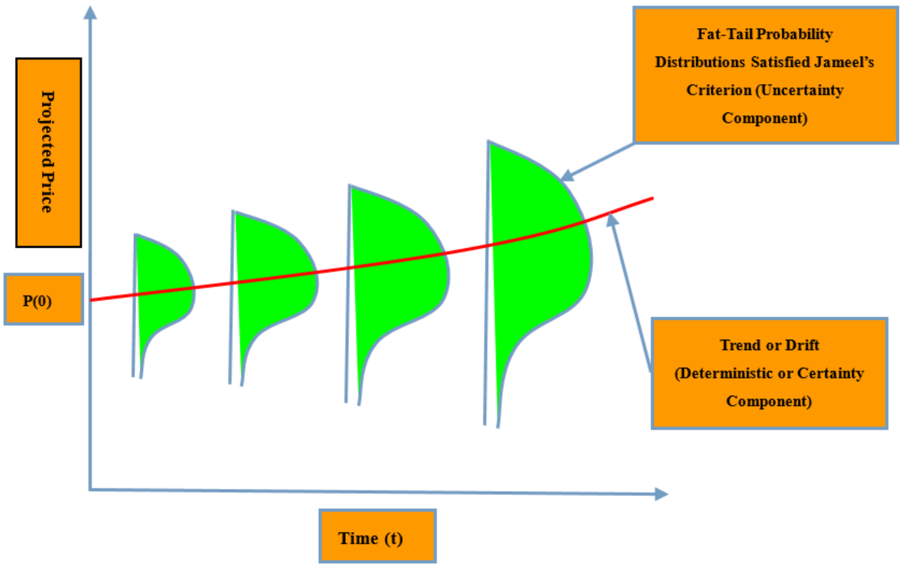

Figure 4. Non-Normal Stocks Brownian Motion

Source: The Author (2017)

2.2.6 Bitcoin Pricing for IFRS 9 Compliance

The Bitcoin Price was modeled as Geometric Fractional Brownian Motion by Biagin, et al., (2008) and Mariusz Tarnopolski (2017) using Monte Carlo Approach and generated large number (10^4) of Fractional Brownian Motion (FBM) realizations B^H with Hurst exponent H, and inserts them into the *Closed Form Solution:*$X(t) = X_0 \exp(\mu t + \sigma B_t^H)$ of the stochastic differential equation describing a geometric fractional Brownian Motion: $dX(t) = \mu X(t)dt + \sigma X(t)dB_t^H$, where $X_0 = X(0)$ is the initial price of the extensions, taken as the last price in the historical data set and μ and σ are the drift and volatility respectively.

2.2.7 Propose Jameel's Stressed Closed Form Bitcoin Pricing Models for IFRS 9 Compliance

The existing *Closed Form Solution* of the BITCOIN is given by:$X(t) = X_0 \exp(\mu t + \sigma B_t^H)$ then using Jameel's Criterion and Jameel's Contractional-Expansional Stressed Methods thereby REPLACING the LOG-NORMAL or NORMAL PROCESS $\{B_t^H\}_{t \geq 0}$ with the JAMEEL'S SUBSTITUTIONS for IFRS 9 : (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, $\mu_A = 0$, where μ_A is POSITIVE INFINITESIMAL, $\sigma_A \geq 1$ and define σ_A as Geometric Volatility ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and μ_A as Geometric Means ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters, $\{W_{JB(t)}\}$ is a Non-Normal Brownian Motion variable fat-tail stochastic probability distribution of the considered Financial Instrument Return Satisfying Jameel's Criterion then we have the following *Propose Jameel's Stressed Closed Form Bitcoin Pricing Models TYPESfor IFRS 9 Compliance as:*

TYPE 1:

$\left(X(t)\right)_{Stressed} = X_0 \exp\left(\mu t + \sigma\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right)$,whenever $\sigma_A > 1$, μ_A is positive infinitesimal;

TYPE 2:

$\left(X(t)\right)_{Stressed} = X_0 \exp\left(\mu t + \sigma\left(\mu_A \pm W_{JB}(t)\right)\right)$,whenever $\sigma_A = 1$, μ_A is positive infinitesimal;

TYPE 3:

$\left(X(t)\right)_{Stressed} = X_0 \exp\left(\mu t + \sigma\left(\pm\sigma_A W_{JB}(t)\right)\right)$,whenever $\sigma_A > 1$, $\mu_A = 0$;

TYPE 4:

$\left(X(t)\right)_{Stressed} = X_0 \exp\left(\mu t + \sigma\left(\pm W_{JB}(t)\right)\right)$,whenever$\sigma_A = 1$, $\mu_A = 0$;

2.2.8 Ornstein – Uhlenbeck Process for IFRS 9 Compliance

Ornstein – Uhlenbeck Process can be used to model Interest Rates, Currency Exchange Rates, and Commodity Prices stochastically. It can also be used in Trading Strategy known as PAIRS TRADE. An Ornstein – Uhlenbeck Process, x_t , satisfies the following stochastic differential equation: $dx_t = \theta(\mu - x_t)dt + \sigma dW_t$, where $\theta > 0$, μ and $\sigma > 0$ are parameters and W_t denotes the Weiner process. μ is the mean value supported by fundamentals, σ is the degree of volatility aroundit caused by shocks, θ is the rate by which these shocks dissipate and variable reverts towards the mean.

The Closed Form Solution of Ornstein – Uhlenbeck Process is given by:

$X_t = \mu + \theta(x_0 - \mu)e^{-\theta t} + \sigma\int_0^t e^{-\theta(t-s)}\, dW_s$. Note that this is a sum of deterministic terms and an integral of a deterministic function with respect to a Wiener process with normally distributed increments. The distribution is thus NORMAL.

2.2.9 Cox-Ingersoll-Ross (1985) Model for IFRS 9 Compliance

Cox-Ingersoll-Ross (1985) Model (CIR) describes Interest Rate Movements as driven by only one source of market risk and Interest Rate Derivatives. Also, under the no-arbitrage assumption, a BOND could be priced using this interest rate process and to Price Default Free Zero-Coupon Bonds. The CIR Model is given by the following stochastic differential equation:

$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW_t$, where W_t is a Wiener Process (modeling the random walk market risk factor) and a, b and σ are the parameters. The parameter a corresponds to the speed of adjustment, b, the mean and σ is the volatility. The drift factor,$a(b - r_t)$ is exactly the same as in the Vasicek Model. It ensures mean reversion of the interest rate towards the long run value, b with speed of adjustment governed by the strictly positive parameter a. The standard deviation factor,$\sigma\sqrt{r_t}$, avoids the possibility of negative interest rates for all positive values of a and b. An interest rate of zero is also precluded if the condition $2ab \geq \sigma^2$ is met.

The *Closed Form Solution* of *Cox-Ingersoll-Ross (1985) Model* is given by:

$$r_t = \theta + (r_0 - \theta)e^{-kt} + \sigma e^{-kt}\int_0^T e^{ku}\sqrt{r_u}\,dW_u$$

2.2.10 Vasicek Model for IFRS 9 Compliance

Vasicek Model can be used to describes Interest Rate Movements and in the Valuation of Interest Rate Derivatives. The model specifies that the instantaneous interest rate follows the stochastic differential equation: $dr_t = a(b - r_t)dt + \sigma dW_t$. Where W_t is a Wiener process under the risk neutral framework modeling the random market risk factor. σis the volatility of the interest rate, b is the long term level, and a is the speed of reversion. The Closed Form Solution of Vasicek Model is given by:

$$r_t = r(0)e^{-at} + b(1 - e^{-at}) + \sigma e^{-at}\int_0^T e^{as}\,dW_s$$

2.2.11 Black-Karasinki (1991) Model for IFRS 9 Compliance

Black-Karasinki (1991) Model can be used for Term Structure of Interest Rates and it can fit Today's Zero-Coupon Bond Prices, and Today's Prices for a set of Caps, Floors, or European Swaptions. It can also be used for Pricing of Exotic Interest Rate Derivatives such as American and Bermudan Bond Options and Swaptions. The main state variable of the model is the short rate, which is assumed to follow the stochastic differential equation (under risk – neutral measure): $n(r) = [\theta_t - \phi\ln(r)]dr + \sigma_t dW_t$, where dW_t is the Standard Brownian Motion. The Model implies a LOG-NORMAL DISTRIBUTION for short rate.

2.2.12 Chen (1994) Model for IFRS 9 Compliance

Chen (1994) Model describes the evolution of Interest Rates.The dynamics of the instantaneous Interest Rate are specified by the stochastic differential equations:

$$dr_t = (\theta_t - \alpha_t)dr + \sqrt{r_t}\sigma_t dW_t$$

$$d\alpha_t = (\xi_t - \alpha_t)dr + \sqrt{\alpha_t}\sigma_t dW_t$$

$$d\sigma_t = (\beta_t - \sigma_t)dr + \sqrt{\sigma_t}\eta_t dW_t$$

2.2.13 Kalotay – Williams – Fabozzi (1993) Model for IFRS 9 Compliance

Kalotay – Williams – Fabozzi (1993) Model describes the dynamics of Short Rate and is given by:

$$dln(r_t) = \theta_t dr + \sigma dW_t$$

2.2.14 Longstaff - Schwatz (1992) Model for IFRS 9 Compliance

Longstaff - Schwatz (1992) Model describes the dynamics of Interest Rate and is given by:

$$dX_t = (a_t - bX_t)dt + \sqrt{X_t}c_t\,dW_{1t}$$

$$dY_t = (d_t - eX_t)dt + \sqrt{Y_t}f_t\,dW_{2t}$$

Where the short rate is defined as:

$dr_t = (\mu X - \theta Y)dt + \sigma_t\sqrt{Y}dW_{3t}$. Where, dW is the Standard Brownian Motion.

2.2.15 Ho-Lee Model (1986) Model for IFRS 9 Compliance

The model return the price of BONDS comprising the yield curve and subsequently can be used in valuation of BOND OPTIONS, SWAPTIONS, and other Interest DERIVATIVES which is typically performed via a binomial lattice based model. It can also be used in closed form valuations of BOND, and "BLACK-LIKE" Bond Option Formulae.The process describe the evolution of the short rate r as follows: $dr = \theta(t)dt + \sigma dz$, where $\theta(t)$ is the expected change, or drift, in the short rate and σdz a stochastic term which models the random component (volatility) of the short rate. The parameter σ is the volatility of the short rate and it is assumed to be constant, that is it does not change with time.

2.2.16 Hull-White (1990) Model for IFRS 9 Compliance

The Hull and White (1990) term structure model extends the Ho-Lee model by incorporating the mean reversion property of interest rates as follows:

$dr = (\theta - \varphi r)dt + \sigma dz$; where the parameters θ and φ are constants. The volatility of the short rate σ is assumed constant across time periods as in the Ho-Lee Model.

2.2.17 Black-Derman-Toy(1990) Model for IFRS 9 Compliance

The Black-Derman-Toy (1990) Model term structure model, unlike the previous models discussed, assumes that the short rate distribution is lognormal instead of normal and is given by: $d\ln(r) = \left[\theta(t) + \frac{\sigma'(t)}{\sigma(t)}.\ln(r)\right]dt + \sigma(t)dz$,

where z_{JB} satisfied Jameel's Criterion, $\sigma(t)$ is the volatility at time t. $\ln(r)$ is the natural logarithm of the short rate, and $\theta(t)$ is the time varying drift parameter.

2.2.18 Heston Volatility Model for IFRS 9 Compliance

Heston Model is a financial model use to describe the evolution of the volatility of an underlying asset. The model assumes that S_t, the price of the asset is determine by a stochastic process: $dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S$, where v_t is the instantaneous variance and given by: $dv_t = k(\theta - v_t)dt + \boldsymbol{\xi}\sqrt{v_t}dW_t^v$ and W_t^S, W_t^v are Wiener Processes with correlation ρ or equivalently, with variance ρdt. Where, μ is the rate of return of the asset, θ is the long variance or long run average price variance as t tends to infinity, the expected value of v_t tends to θ, k is the rate at which v_t reverts to θ, ξ is the volatility of the volatility or Vol of Vol and determines the variance of v_t. If the parameters obey the following condition (known as the feller condition) then the process v_t is strictly positive that $2k\theta > \xi^2$.

2.2.19 ETFs and Leveraged ETFs Pricing for IFRS 9 Compliance

The dynamics of ETF using stochastic calculus can be written as:

ETF: $S_t = \exp(X_t(x))$, $X_t(x) = x + \int_0^t \mu(X_s)ds + \int_0^t \sigma(X_s)dW_s + \int_0^t \int_{\mathbb{R}_0} Z\bar{N}(ds, dz)$.

While, the evolution of the underlying index $(S_t)_{t\geq 0}$ of *Leveraged ETF is given by a Geometric Brownian Motion (GBM)*: $dS_t = S_t(\mu dt + \sigma dW_t)$, where W is a standard Brownian Motion under the historical measure. μ is the ex-dividend annualized growth rate and $\sigma > 0$ is the constant volatility. Thus, the dynamics of *Leveraged ETF* using stochastic calculus can be written as: LETF: $L_t = 1_{\{T>t\}}\exp(Y_t(x))$, $Y_t(x) = x + \int_0^t \alpha(X_s)ds + \beta\int_0^t \sigma(X_s)dW_s + \int_0^t\int_{A_0} \mu B(z)\bar{N}(ds, dz)$, for $t \geq 0$, where, $\mu(u) := -\frac{1}{2}\sigma^2(u) - \int_{\mathbb{R}_0}(e^z - 1 - z)v(dz)$,

$\alpha(u) := V(A^c) - \frac{1}{2}\beta^2\sigma^2(u) - \int_{A_0}[\beta(e^z - 1) - u\beta(z)]v(dz)$.

2.2.20 Propose Jameel's Stressed Closed FormModels presented from 2.2.8 to 2.2.19. for IFRS 9 Compliance

Generally, using Jameel's Criterion and Jameel's Contractional-Expansional Stressed Methods, we replaces the WIENER PROCESSES (NORMAL and or LOG-NORMAL) terms appears in the CLOSED FORM SOLUTIONS of *Ornstein – Uhlenbeck Process, Cox-Ingersoll-Ross (1985) Model, Vasicek Model, Black-Karasinki (1991) Model, Chen (1994) Model, Kalotay – Williams – Fabozzi (1993) Model, Longstaff - Schwatz (1992) Model, Ho-*

Lee Model (1986) Model, Hull-White (1990) Model, Black-Derman-Toy(1990) Model, Heston Volatility Model andETFs and Leveraged ETFs Models by JAMEEL'S SUBSTITUTIONS FOR IFRS 9 COMPLIANCE as : (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if $\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if $\sigma_A = 1$, $\mu_A = 0$, where μ_A is POSITIVE INFINITESIMAL, $\sigma_A \geq 1$ and define σ_A as Geometric Volatility ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and μ_A as Geometric Means ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters, $\{W_{JB(t)}\}$ is a Non-Normal Brownian Motion variable fat-tail stochastic probability distribution of the considered Financial Instrument Return Satisfying Jameel's Criterion to obtain their *Stressed Closed Form Models TYPESfor IFRS 9 Compliance.*

3. Results

To test the performances of the proposed Sixteen (16) Jameel's Stressed Closed Form Solutions considering Stocks Geometric Brownian Model, the Author considered Chevron Corporation (CVX) Stock data extracted from yahoo finance using Time Series from 2014 – 1991. Thus, the data distribution Mean equal 0.000326, Standard Deviation equal 0.015761, the Annual drift of the year preceding 2014 (2013) equal 0.000466 and the Annual Volatility of the year preceding 2014 (2013) equal 0.008325. Hence, $\mu_{daily} = 0.000466/252 = 1.84921E-06$, $\sigma_{daily} = 0.008325/\sqrt{252} = 0.000524$.

Therefore, $\mu = \mu_{daily} - \frac{1}{2}\sigma_{daily}^2 = 1.84921E-06 - \frac{1}{2}(0.000524)^2 = 1.71192E-06$ and $\sigma = 0.000524$

while $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$.

Table 1. CVX Stressed Prices with Log-Logistic (3P) compared with Real and Normal Prices

Date	t	REAL PRICES	NORMAL PRICES	LL(3P) T1+	LL(3P) T1-	LL(3P) T2+	LL(3P) T2-	LL(3P) T3+	LL(3P) T3-	LL(3P) T4+	LL(3P) T4-
11/28/2014	**0**	**108.87**									
12/1/2014	1	111.730003	108.8701864	108.8719768	108.8718627	108.8724317	108.8714077	108.8702434	108.8701293	108.8706984	108.8696744
12/2/2014	2	114.019997	111.7303855	111.7322223	111.7321066	111.7326839	111.731645	111.7304434	111.7303277	111.730905	111.7298661
12/3/2014	3	113.709999	114.0205826	114.0224565	114.0223394	114.0229233	114.0218726	114.0206411	114.0205241	114.0211079	114.0200573
12/4/2014	4	112.279999	113.7107777	113.7126465	113.7125296	113.7131126	113.7120635	113.7108361	113.7107192	113.7113022	113.7102531
12/5/2014	5	110.870003	112.2809601	112.2828058	112.2826897	112.2832686	112.2822269	112.2810181	112.280902	112.2814809	112.2804392
12/8/2014	6	106.800003	110.8711418	110.8729647	110.8728494	110.8734242	110.8723898	110.8711994	110.8710842	110.871659	110.8706246
12/9/2014	7	107.010002	106.8012828	106.8030397	106.8029268	106.8034898	106.8024767	106.8013393	106.8012264	106.8017894	106.8007763
12/10/2014	8	104.860001	107.0114676	107.0132278	107.0131148	107.0136784	107.0126642	107.011524	107.0114111	107.0119746	107.0109605
12/11/2014	9	104.910004	104.8616166	104.863342	104.8632303	104.8637875	104.8627848	104.8616725	104.8615608	104.862118	104.8611152
12/12/2014	10	102.379997	104.9118	104.9135262	104.9134144	104.9139718	104.9129688	104.9118559	104.9117441	104.9123015	104.9112985
12/15/2014	11	100.860001	102.3819249	102.3836101	102.3834999	102.3840497	102.3830603	102.3819801	102.3818698	102.3824196	102.3814303
12/16/2014	12	101.699997	100.862073	100.8637335	100.8636242	100.8641694	100.8631883	100.8621276	100.8620183	100.8625636	100.8615824
12/17/2014	13	106.019997	101.7022604	101.7039345	101.7038247	101.7043725	101.7033867	101.7023153	101.7022054	101.7027532	101.7017675
12/18/2014	14	109.029999	106.022538	106.0242822	106.0241698	106.0247305	106.0237215	106.0225942	106.0224818	106.0230425	106.0220335
12/19/2014	15	112.93	109.0327988	109.0345918	109.0344777	109.0350472	109.0340223	109.0328559	109.0327417	109.0333112	109.0322864
12/22/2014	16	112.029999	112.9330933	112.9349495	112.9348331	112.9354139	112.9343688	112.9331515	112.9330351	112.9336158	112.9325707
12/23/2014	17	113.949997	112.0332594	112.0351011	112.0349852	112.0355634	112.0345229	112.0333174	112.0332015	112.0337796	112.0327392
12/24/2014	18	113.470001	113.9535084	113.9553812	113.9552642	113.9558478	113.9547975	113.9535669	113.9534499	113.9540335	113.9529832
12/26/2014	19	113.25	113.4736918	113.4755569	113.4754401	113.4760224	113.4749746	113.4737502	113.4736335	113.4742158	113.4731679
12/29/2014	20	113.32	113.2538776	113.255739	113.2556224	113.2562041	113.2551574	113.2539359	113.2538193	113.2544009	113.2533542
12/30/2014	21	113.110001	113.324074	113.3259366	113.3258199	113.3264018	113.3253547	113.3241323	113.3240156	113.3245975	113.3235504

The Author uses $p_0 = 108.87$ as of 11/28/2014 (a day before 12/1/2014) as the *Initial Stock Price* with intention to *Predict Twenty One (21) working days (from 12/1/ 2014 to 12/30/ 2014) Chevron Corporation (CVX) Stock Prices* thereby comparing the REAL PRICES, NORMAL PRICES with that ofSIXTEEN (16) PROPOSED JAMEEL'S STRESSED CLOSED FORM PRICESusing Top two and 4th fat-tailed Non-Normal Probability Distribution Functions satisfied Jameel's Criterion and are thus: LOG-LOGISTIC (3P), CAUCHY and BURR(4P).

The Author performs the PREDICTION Using MICROSOFT EXCEL and obtained the following RESULTS as shown in Tables and Charts below:

Note that in Table 1, the notation LL (3P) $\boldsymbol{T_i}(+\ \textbf{or}\ -), \boldsymbol{i} = \mathbf{1, 2, 3, 4}$means Positive or Negative Jameel's Stressed Closed Form Prices TYPES 1 to 4 with respect to LOG-LOGISTIC (3P), in Table 2, the notation Cauchy $\boldsymbol{T_i}(+\ \textbf{or}\ -), \boldsymbol{i} = \mathbf{1, 2, 3, 4}$means Positive or Negative Jameel's Stressed Closed Form Prices TYPES 1 to 4 with respect to CAUCHY, InTable 3, the notation Burr (4P) $\boldsymbol{T_i}(+\ \textbf{or}\ -), \boldsymbol{i} = \mathbf{1, 2, 3, 4}$means Positive or Negative Jameel's Stressed Closed Form Prices TYPES 1 to 4 with respect to BURR (4P).

Table 2. CVX Stressed Prices with Cauchy compared with Real and Normal Prices

Date	t	REAL PRICES	NORMAL PRICES	Cauchy T1+	Cauchy T1-	Cauchy T2+	Cauchy T2-	Cauchy T3+	Cauchy T3-	Cauchy T4+	Cauchy T4-
	0	108.87									
12/1/2014	1	111.730003	108.8701864	108.8719197	108.8719197	108.8719197	108.8719197	108.8701864	108.8701864	108.8701864	108.8701864
12/2/2014	2	114.019997	111.7303855	111.7321644	111.7321644	111.7321644	111.7321644	111.7303855	111.7303855	111.7303855	111.7303855
12/3/2014	3	113.709999	114.0205826	114.0223979	114.0223979	114.0223979	114.0223979	114.0205826	114.0205826	114.0205826	114.0205826
12/4/2014	4	112.279999	113.7107777	113.7125881	113.7125881	113.7125881	113.7125881	113.7107777	113.7107777	113.7107777	113.7107777
12/5/2014	5	110.870003	112.2809601	112.2827477	112.2827477	112.2827477	112.2827477	112.2809601	112.2809601	112.2809601	112.2809601
12/8/2014	6	106.800003	110.8711418	110.872907	110.872907	110.872907	110.872907	110.8711418	110.8711418	110.8711418	110.8711418
12/9/2014	7	107.010002	106.8012828	106.8029833	106.8029833	106.8029833	106.8029833	106.8012828	106.8012828	106.8012828	106.8012828
12/10/2014	8	104.860001	107.0114676	107.0131713	107.0131713	107.0131713	107.0131713	107.0114676	107.0114676	107.0114676	107.0114676
12/11/2014	9	104.910004	104.8616166	104.8632862	104.8632862	104.8632862	104.8632862	104.8616166	104.8616166	104.8616166	104.8616166
12/12/2014	10	102.379997	104.9118	104.9134703	104.9134703	104.9134703	104.9134703	104.9118	104.9118	104.9118	104.9118
12/15/2014	11	100.860001	102.3819249	102.383555	102.383555	102.383555	102.383555	102.3819249	102.3819249	102.3819249	102.3819249
12/16/2014	12	101.699997	100.862073	100.8636789	100.8636789	100.8636789	100.8636789	100.862073	100.862073	100.862073	100.862073
12/17/2014	13	106.019997	101.7022604	101.7038796	101.7038796	101.7038796	101.7038796	101.7022604	101.7022604	101.7022604	101.7022604
12/18/2014	14	109.029999	106.022538	106.024226	106.024226	106.024226	106.024226	106.022538	106.022538	106.022538	106.022538
12/19/2014	15	112.93	109.0327988	109.0345347	109.0345347	109.0345347	109.0345347	109.0327988	109.0327988	109.0327988	109.0327988
12/22/2014	16	112.029999	112.9330933	112.9348913	112.9348913	112.9348913	112.9348913	112.9330933	112.9330933	112.9330933	112.9330933
12/23/2014	17	113.949997	112.0332594	112.0350431	112.0350431	112.0350431	112.0350431	112.0332594	112.0332594	112.0332594	112.0332594
12/24/2014	18	113.470001	113.9535084	113.9553227	113.9553227	113.9553227	113.9553227	113.9535084	113.9535084	113.9535084	113.9535084
12/26/2014	19	113.25	113.4736918	113.4754985	113.4754985	113.4754985	113.4754985	113.4736918	113.4736918	113.4736918	113.4736918
12/29/2014	20	113.32	113.2538776	113.2556807	113.2556807	113.2556807	113.2556807	113.2538776	113.2538776	113.2538776	113.2538776
12/30/2014	21	113.110001	113.324074	113.3258782	113.3258782	113.3258782	113.3258782	113.324074	113.324074	113.324074	113.324074

Table 3. CVX Stressed Prices with Burr (4P) compared with Real and Normal Prices

Date	t	REAL PRICES	NORMAL PRICES	Burr (4P) T1+	Burr (4P) T1-	Burr (4P) T2+	Burr (4P) T2-	Burr (4P) T3+	Burr (4P) T3-	Burr (4P) T4+	Burr (4P) T4-
	0	108.87									
12/1/2014	1	111.730003	108.8701864	108.8719197	108.8719197	108.8719198	108.8719197	108.8701864	108.8701864	108.8701864	108.8701863
12/2/2014	2	114.019997	111.7303855	111.7321644	111.7321644	111.7321645	111.7321644	111.7303856	111.7303855	111.7303856	111.7303855
12/3/2014	3	113.709999	114.0205826	114.0223979	114.0223979	114.022398	114.0223979	114.0205826	114.0205826	114.0205826	114.0205825
12/4/2014	4	112.279999	113.7107777	113.7125881	113.7125881	113.7125881	113.712588	113.7107777	113.7107776	113.7107777	113.7107776
12/5/2014	5	110.870003	112.2809601	112.2827477	112.2827477	112.2827478	112.2827477	112.2809601	112.2809601	112.2809601	112.28096
12/8/2014	6	106.800003	110.8711418	110.872907	110.872907	110.8729071	110.872907	110.8711418	110.8711418	110.8711419	110.8711418
12/9/2014	7	107.010002	106.8012828	106.8029833	106.8029833	106.8029833	106.8029832	106.8012828	106.8012828	106.8012829	106.8012828
12/10/2014	8	104.860001	107.0114676	107.0131713	107.0131713	107.0131714	107.0131713	107.0114676	107.0114675	107.0114676	107.0114675
12/11/2014	9	104.910004	104.8616166	104.8632862	104.8632862	104.8632862	104.8632861	104.8616166	104.8616166	104.8616167	104.8616166

12/12/2014	10	102.379997	104.9118	104.9134703	104.9134703	104.9134704	104.9134703	104.9118	104.9118	104.9118	104.9117999
12/15/2014	11	100.860001	102.3819249	102.383555	102.383555	102.3835551	102.383555	102.381925	102.3819249	102.381925	102.3819249
12/16/2014	12	101.699997	100.862073	100.8636789	100.8636788	100.8636789	100.8636788	100.862073	100.862073	100.862073	100.8620729
12/17/2014	13	106.019997	101.7022604	101.7038796	101.7038796	101.7038796	101.7038795	101.7022604	101.7022603	101.7022604	101.7022603
12/18/2014	14	109.029999	106.022538	106.024226	106.024226	106.0242261	106.024226	106.022538	106.022538	106.022538	106.022538
12/19/2014	15	112.93	109.0327988	109.0345347	109.0345347	109.0345348	109.0345347	109.0327988	109.0327988	109.0327988	109.0327988
12/22/2014	16	112.029999	112.9330933	112.9348913	112.9348913	112.9348914	112.9348913	112.9330933	112.9330933	112.9330933	112.9330932
12/23/2014	17	113.949997	112.0332594	112.0350431	112.0350431	112.0350432	112.0350431	112.0332594	112.0332594	112.0332595	112.0332594
12/24/2014	18	113.470001	113.9535084	113.9553227	113.9553227	113.9553227	113.9553226	113.9535084	113.9535084	113.9535084	113.9535083
12/26/2014	19	113.25	113.4736918	113.4754985	113.4754985	113.4754985	113.4754985	113.4736918	113.4736918	113.4736919	113.4736918
12/29/2014	20	113.32	113.2538776	113.2556807	113.2556807	113.2556808	113.2556807	113.2538776	113.2538776	113.2538776	113.2538775
12/30/2014	21	113.110001	113.324074	113.3258782	113.3258782	113.3258783	113.3258782	113.324074	113.324074	113.324074	113.3240739

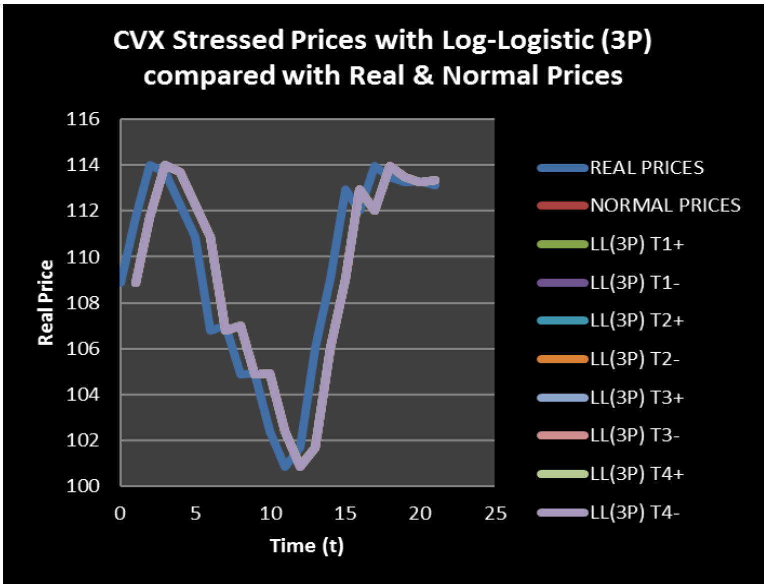

Figure 5.

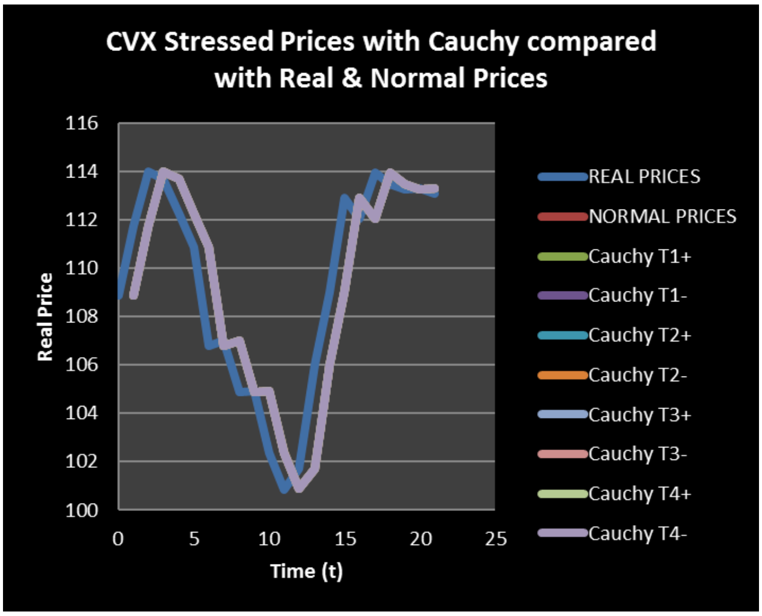

Figure 6.

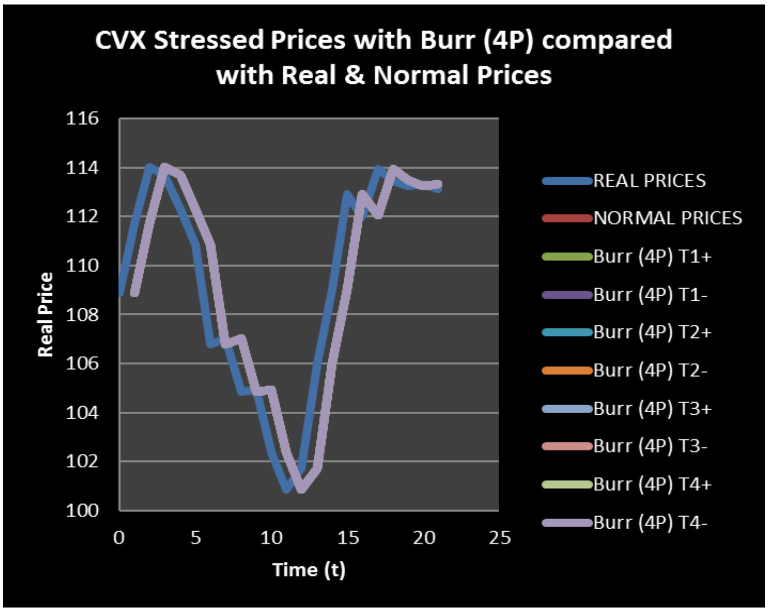

Figure 7.

It can be observed all the ***Shaded Areas in Table 1 to 3*** approximated or almost coincided with the Chevron Corporation **REAL PRICES**. While ***figure 5 to 7*** shows the performances of the proposed SIXTEEN (16) JAMEEL'S STRESSED CLOSED FORM PRICES vis-à-vis REAL PRICES and NORMAL PRICES.

The results performances were FASCINATINGLY interesting, impressive, viable, reliable, sophisticated and complaint with IFRS 9 since they ***incorporated the forward-looking information*** $\{\boldsymbol{W_{JB}(t)}\}$satisfying Jameel's Criterion and Geometric average of only positive ***Economic forecasts of the future Macroeconomic scenarios*** $\{(\mu_A)\ \boldsymbol{and}(\boldsymbol{\sigma_A})\}$ and also minimized the differences between Market Prices and Model Prices of the Financial Instruments.

4. Discussion

In this paper, the performances of the SIXTEEN (16) PROPOSED JAMEEL'S STRESSED CLOSED FORM MODELS with respect to LOG-LOGISTIC (3P), CAUCHY, and BURR (4P) can be improved using the following:

1) Accurate prediction of economic forecasts of fundamental macroeconomic parameters used in the proposed models

2) The Author set the Log-Logistic (3P) parameter **ξ** to be 1 and Burr (4P) parameters $a = 1, k = 1, \gamma = 1, \beta = 1\ and\ \alpha = 2$ thus collapsed to almost Normal. With HIGH VALUES of **ξ**, a, k, γ, β, and α, the proposed Jameel's Stressed Closed Prices will effectively approximates the REAL PRICES.

3) Jameel's Criterion axiom known as **"Criterion Enhancement Axiom"** : That if we could be able to Runs the Goodness of Fit Tests such as the RANKS of Kolmogorov Smirnov (KS) Test, Anderson-Darling Test, Jarque and Bera (JB) Test, Shapiro Wilk (SW) Test, Cramer-Von Mises Test, Pearson $\left(\chi^2\ Godness\ of\ Fit\right)$ Test, Lilliefors Corrected K-S Test, D'AgostinoSkewness Test, Anscombe-Glynn Kurtosis Test, D'Agostino-Pearson Omnibusare all **UNITY (1)** of the underlying Stock Returns then the proposed Jameel's Stressed Closed Prices will coincide at finitely many points with the REAL PRICES .

4) $\mu_A$can be **TESTED** as **ARITHMETIC** Means ofonly positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters, otherwise should remains **GEOMETRIC MEANS** as defined and used in the paper.

Finally, the results performances ofthe PROPOSED JAMEEL'S STRESSED CLOSED FORM SOLUTIONS of Ornstein – Uhlenbeck Process, Cox-Ingersoll-Ross (1985) Model, Vasicek Model, Black-Karasinki (1991) Model, Chen (1994) Model, Kalotay – Williams – Fabozzi (1993) Model, Longstaff - Schwatz (1992) Model, Ho-Lee Model (1986) Model, Hull-White (1990) Model, Black-Derman-Toy(1990) Model, Heston Volatility Model andETFs and Leveraged ETFs Models can be TESTED using the processes as in the case of STOCKS. Also, the models would provide excellent results using MONTE-CARLO or GENERAL SIMULATION ANALYSES.

5. Conclusion

There are three major pillars of IFRS 9, basically; Classification and Measurement of Financial Instruments, Impairment and Hedge Accounting. Measurement of Financial Instruments is the most challenging aspect, because it requires sophisticated Credit Risk Modeling Skills and Expertise. According to IFRS 9, *"The Credit Risk at origination is included in the pricing of Financial Asset but any increase in Credit Risk is NOT".* Refreshing, improving, or updating the existing Financial Instruments PRICING MODELS is seriously needed through the incorporation of forward-looking information and economic forecasts of the future macroeconomic scenarios.

The existing Financial Instruments Pricing Models either were built using the ideas of BROWNIAN MOTION, FRACTIONAL BROWNIAN MOTION or simply MIXTURE of former, later or both former and later. However, both of the Brownian and Fractional Brownian Motions usually collapsed to LOG-NORMAL or NORMAL probability distribution pricing modeling TRENDS which simply UNDERESTIMATES/OVERESTIMATES financial instruments' MARKET PRICES.

The paper uses Jameel's Criterion and Jameel's Contractional-Expansional Stress Methods thereby REPLACING the WEINER PROCESS $\{W(t)\}_{t\geq 0}$ by the JAMEEL'S SUBSTITUTIONS : (i) $\left\{\left(\mu_A \pm \sigma_A W_{JB}(t)\right)\right\}$, if$\sigma_A > 1$, μ_A is positive infinitesimal; (ii) $\left\{\left(\mu_A \pm W_{JB}(t)\right)\right\}$, if$\sigma_A = 1$, μ_A is positive infinitesimal; (iii) $\left\{\left(\pm\sigma_A W_{JB}(t)\right)\right\}$, if$\sigma_A > 1$, $\mu_A = 0$, and; (iv) $\left\{\left(\pm W_{JB}(t)\right)\right\}$, if$\sigma_A = 1$, $\mu_A = 0$ into the CLOSED FORM SOLUTIONS of the existing Geometric Brownian Motion (1920),Cox-Ingersoll-Ross (1985),Ornstein-Uhlenbeck (1930), Vasicek (1977), Black-Karasinki (1991), Chen (1994), Kalotay-Williams-Fabozzi (1993), Longstaff-Schwatz (1992), Ho-Lee (1986), Hull and White (1990), and Black-Derman-Toy (1990), Biagin, et al.,(2008)Models for Pricing Stocks, ETFs, and Leveraged ETFs, Bonds, Bitcoin, Interest Rate Movements, Caps, Floors, European Swaptions and Bond Options.

Finally, the STRESSED CLOSED FORM SOLUTIONS of Jameel's based Stocks Brownian Motion Model was tested, the results performances were FASCINATINGLYinteresting, impressive, viable, reliable, sophisticated and complaint with IFRS 9 since they ***incorporated the forward-looking information*** $\{\boldsymbol{W_{JB(t)}}\}$satisfying Jameel's Criterion and Geometric average of only positive ***Economic forecasts of the future Macroeconomic scenarios*** $\{(\mu_{\boldsymbol{A}})\ \boldsymbol{and}(\boldsymbol{\sigma_A})\}$and minimized the differences between Market Prices and Model Prices of the Financial Instruments.

To crown it up, Jameel's Advanced Stressed Models for IFRS 9 Compliance can be summarized as follows: (i) They minimized the difference between MARKET PRICES and MODELS PRICES of FINANCIAL INSTRUMENTS thereby incorporating forward-looking information $\{W_{JB(t)}\}$ satisfies Jameel's Criterion and Geometric average of only positive Economic forecasts of the future Macroeconomic scenarios $\{(\mu_A)\ \mathrm{and}(\sigma_A)\}$ and, (ii) They captured the impact of Low-Probability, High-Impact Events in the Default Probability Models and Expected Credit Risk Loss Models uses for the calculations of Banks Capital and Provisioning Numbers.

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