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Jameel's Dimensional Stressed Default Probability Models are Indeed IFRS 9 Complaint Models

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Abstract

Default Probabilities can be used to Analysis Firm Creditworthiness, calculate Expected Credit Losses, Economic and Regulatory Capitals for Banking Institutions and Ranking such as FICO for consumers or Bond Ratings from S & P, Fitch or Moodys for Corporations and Governments. Banks and other Institutions are heavily investing towards Building, Developing, Improving and or Purchasing Credit Risk Models that would enhance their capabilities to handle, predict and quantify Credit Risk challenges which will subsequently help them to accurately calculate and assign sufficient Economic and Regulatory Capitals. It is believe that the existing Default Probability Models failed to accurately predict the unforeseen level of borrower defaults and resulting losses they had to recognize. The IASB in July, 2014 issued the final version of IFRS 9 Measurement of Financial Instruments beginning on or after 1st January, 2018 with early adoption permitted. It replaces IAS 39 Financial Instruments: Recognition and Measurement. IFRS 9 require adjustments to the use of Probability of Default (PD), Exposure at Default and Loss Given Default (LGD) estimates. Probability of Default (PD) plays very important role when Calculating Expected Credit Losses under IFRS 9. Jamilu (2015) enhanced LOGIT and PROBIT Default Probability Models with the aid of one-dimensional forward-looking information $\{f(x)\}$ satisfies Jameel's Criterion and positive average of economic forecasts of future macroeconomic scenarios $\{\mu_A > 0, infinitesimal\ and\ \sigma_A \geq 1\}$. This paper further enhance LOGIT and PROBIT Default Probability Models using Two and Three Dimensional Forward-Looking Information(s) $\{f_1(x), f_2(x)\}$ and $\{f_1(x), f_2(x), f_3(x)\}$ respectively satisfies Jameel's Criterion with LOG-LOGISTIC (3P) $\equiv f_1(x)$, CAUCHY $\equiv f_2(x)$ and BURR(4P) $\equiv f_3(x)$ and positive average of economic forecasts of future macroeconomic scenarios $\{\mu_A\ and\ \sigma_A\}$. The paper tested the performances of only proposed Default Probability Models of TYPES 1 in each class using Twenty One (21) working days (from 12/1/ 2014 to 12/30/ 2014). The results were fascinatingly interesting, impressive, viable, reliable, sophisticated, and complaint with IFRS 9 since they incorporated forward-looking information(s) and Economic forecasts of the future macroeconomic scenarios thereby minimizing the differences between MODELS DEFAULT PROBABILITIES and REAL LIFE DEFAULT PROBABILITIES.

Keywords: Logit, Probit, Probability, Forward-Looking Information, Macroeconomic scenarios, Jameel's Criterion

JEL Classification: A1, C1, C5, C6, E6

1. Introduction

In July, 2014 IASB issued the final version of IFRS 9 Measurement of Financial Instruments beginning on or after 1st January, 2018 with early adoption permitted. It replaces IAS 39 Financial Instruments: Recognition and Measurement. The major target of accounting standards is to provide financial information that stake-holders would find useful when making decisions. The most challenging aspects required by IFRS 9 are the treatment on incorporation of forward-looking information and economic forecasts of future macroeconomic scenarios into the existing Default Probability Models. The IFRS 9 accounting rules regarding Measurement of Financial Instruments will NORROW the wide gaps between **Models Default Probabilities**and **Real Life Default Probabilities**.

Probability of Default can be used to Analysis Firm Creditworthiness, Credit-adjusted Valuation, Economic Capital Calculations, Cash Flow and net income Analyses of firm's obligations, Ranking Firms with the same agency Credit Rating based on estimated default probabilities, Capital Provisioning, Expected Credit Losses, Economic and Regulatory Capital for Banking Institution. Barnaby Black, ShirishChinchalkar, Juan M. Licari

(2016) argued that Regulatory Stress Testing requires that the models should demonstrate sensitivity to macroeconomic conditions.

In response to the credit crisis of 2007-2008, the banking sector adopted international financial regulations to lessen their exposure to default risk. The Basel Committee on Banking Supervision's (BCBS) Goal is to improve the existing banking sector's strategies, processes and ability to deal with FINANCIAL STRESS effectively. Under IFRS 9 Credit Risk Modelling, IFRS 9 reason was that ***"… the Credit Risk at Origination is included in the Pricing of Financial Asset but any increase in Credit Risk is NOT".***

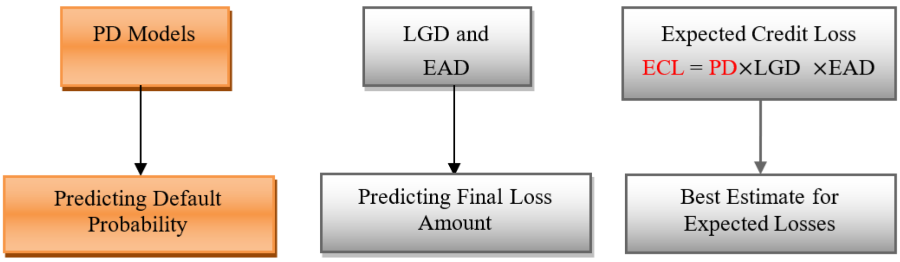
It is believe that the existing Default Probability Models failed to accurately predict the unforeseen level of borrower defaults and resulting losses they had to recognize. Financial Institutions are devoting serious amount of time, energy and resources towards building, developing and purchasing CREDIT RISK MODELS that may improve their abilities to predict and quantify CREDIT RISKS they faces. These credit risk models can adequately improve their abilities to sufficiently calculate Economic and Regulatory Capital reserves.These efforts have been recognized and promoted by Bank Regulators and their Macro prudential Policies.

To address Bankers and Regulators late complain that ***"If only we had seen this coming or had been better prepared…"*** Jamilu (2015) used Jameel's Contractional-Expansional Stressed Methods and Jameel's Criterion to CAME UP with Advanced Stressed Models capable of capturing IFRS 9 INCREASE IN CREDIT RISK that is NOT included at the origination in the PRICING of Financial Assets, Derivatives and Expected Credit Losses (ECLs) components (PD, EAD, LGD) as argued under IFRS 9 Credit Risk Modelling. The objectives of this paper is to further enhance LOGIT and PROBIT Default Probability Models using Two and Three Dimensional Forward-Looking Information(s) $\{f_1(x), f_2(x)\}$ and $\{f_1(x), f_2(x), f_3(x)\}$ respectively satisfies Jameel's Criterion with LOG-LOGISTIC (3P) $\equiv f_1(x)$, CAUCHY $\equiv f_2(x)$ and BURR(4P) $\equiv f_3(x)$ and positive average of economic forecasts of future macroeconomic scenarios$\{\mu_A > 0, Infinitesimal\ and\ \sigma_A \geq 1\}$.

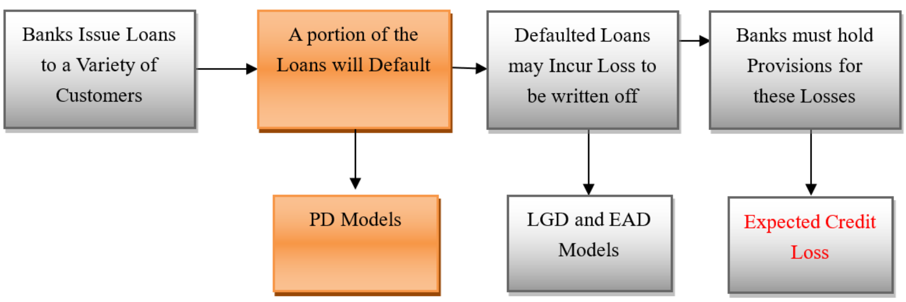
2. Materials and Methods

2.1 Materials

2.1.1 International Financial Reporting Standards 9 (Ifrs 9)



IFRS 9 require adjustments to the use of Probability of Default (PD), Exposure at Default and Loss Given Default (LGD) estimates. As from the above figure, Probability of Default (PD) plays very important role when Calculating Expected Credit Losses under IFRS 9.



The modelling approach for the key risk parameters will be affected by the incorporation of forward – looking, credible and robust economic scenarios into ACCOUNTING MODELS. Banks faces number of challenges in meeting their designed level of IFRS 9 requirements for instance SOPHASTICATED MODELLING EXPECTATIONS, CORRECT MODELS, PEOPLES and SKILLS.

2.1.2 Probability of Default (Pd) or Default Probability

This can be defined as a term describing the likelihood of a Default over a particular time horizon. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. PD is used in a different Credit Analyses and Risk Management Frameworks.

2.1.3 Jameel'S Criterion

Under this criterion, we run the goodness of fits test such that:

i. We accept if the Average of the ranks of Kolmogorov Smirnor, Anderson Darling and Chi-squared is less than or equal to Three (3)

ii. We must choose the Probability Distribution follows by the data **ITSELF** regardless of its Rankings

iii. If there is tie, we include both the Probability Distributions in the selection

iv. At least Two (2) Probability Distributions must be included in the selection

v. We select the most occur Probability Distribution as the qualify candidate in each case of test of goodness of fit.

vi. **Criterion Enhancement Axiom:**Thode (2012) intensively discussed about the Best Goodness of Fit Tests such as Kolmogorov Smirnov (KS) Test, Anderson-Darling Test, Jarque and Bera (JB) Test, Shapiro Wilk (SW) Test, Cramer-Von Mises Test, Pearson $(\chi^2 \text{ Godness of Fit})$ Test, Lilliefors Corrected K-S Test, D'AgostinoSkewness Test, Anscombe-Glynn Kurtosis Test, D'Agostino-Pearson Omnibus Test. Let $\{T_1, T_2, ..., T_n\}$ be the set of such Best Goodness of Fit Tests, $\{x_1, x_2, ..., x_n\}$ be their **RANKS** respectively then the generality of (i) can be expressed (or enhanced) if $\frac{(x_1 + x_2 + ... + x_n)}{n} \leq a$, where $0 < a \leq n,\ n \in N$ or equivalently, $x_1 + x_2 + ... + x_n \leq an$.

vii. **Last Unit Axiom:** let $W_{JB}(t)$ be such that it satisfied axioms (i) to (iv). Let $\{r_1, r_2, ..., r_n\}$ be the ranks of fitness test of $W_{JB}(t)$ obtained from the tests $\{T_1, T_2, ..., T_n\}$ respectively then if $\forall i \in \{1, 2, ..., n\},\ r_i = 1$ regardless of the Time Series, Company and so on. Consequently, if for all fitness test runs, turn out to be the same $W_{JB}(t)$ then the **PREDICTED PRICE PATH** will finitely coincides many times with the **REAL PRICE PATH** of the stock under consideration.

2.2 Methods

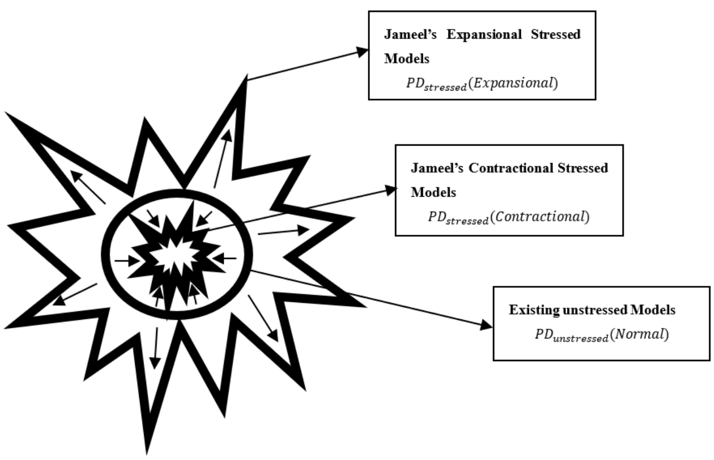


Figure 1. Jameel'sContractional-Expansional Stressed Methods

From figure 1, the basic Idea was to initially use Jameel's Contractional-Expansional Stress Methods to incorporate Low-Probability, High-Impact into the Default Probability Models: **LOGIT** and **PROBIT** Models using

Geometric Volatility σ_A and Geometric Return μ_A of the Arithmetic Means of the underlying Asset Return plus Returns of the explained (independent) variables as well as Jameel's Criterion based Best fitted fat-tailed Probability Distribution of the underlying Asset Return $f(x, \mu_{company}, \sigma_{company}, \xi)$ as worked out below:

- **SHRINKING** the **NORMAL** Probability of Default $PD_{unstressed}(Normal)$ to **CONTRACTIONAL** Probability of Default $PD_{stressed}(Contractional)$ using economic forecasts of future macroeconomic scenarios of Geometric Volatility $\sigma_A \geq 1$ and Only positive Geometric Return $\mu_A > 0$, infinitesimal of the Arithmetic Means of the underlying Asset Return plus Returns of the future macroeconomicparameters as well as ***Jameel's Criterion Best fitted fat-tailed*** forward-looking information $\{f_1(x), f_2(x)\}$ for Two Dimensional and $\{f_1(x), f_2(x), f_3(x)\}$ for Three Dimensional of the underlying Asset Return, where $f_1(x), f_2(x)\ and\ f_3(x)$ are 1st, 2nd and 3rdDistributions Ranking according to Jameel's Criterion.
- **BLOWING** the **NORMAL** Probability of Default $PD_{unstressed}(Normal)$ to **EXPANSIONAL** Probability of Default $PD_{stressed}(Expansional)$ using economic forecasts of future macroeconomic scenarios of Geometric Volatility $\sigma_A \geq 1$ and Only positive Geometric Return $\mu_A > 0$, infinitesimal of the Arithmetic Means of the underlying Asset Return plus Returns of the future macroeconomic parameters as well as ***Jameel's Criterion Best fitted fat-tailed*** forward-looking information $\{f_1(x), f_2(x)\}$ for Two Dimensional and $\{f_1(x), f_2(x), f_3(x)\}$ for Three Dimensional of the underlying Asset Return, where $f_1(x), f_2(x)\ and\ f_3(x)$ are 1st, 2nd and 3rd Distributions Ranking according to Jameel's Criterion.

2.2.1 Logit Default Probability Model

$$PD = \frac{1}{1 + \exp\left(\sum_{i=0}^{K} \beta_i X_i\right)}$$

PD is the probability of default. $X = (X_1, X_2, ..., X_k)$ is a vector of explanatory variables (Macro-economic Indicators).

2.2.2 Probit Default Probability Model

$$PD = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)$$

2.2.3 Propose Two-Dimensional Stressed Logit Default Probability Models

TYPE 1:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm \sigma_A f_2(x)}$$

TYPE 2:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm \sigma_A f_2(x)}$$

TYPE 3:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm f_2(x)}$$

TYPE 4:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm f_2(x)}$$

TYPE 5:

$$PD_{Stressed} = \frac{1}{1 + \exp\left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm \sigma_A f_2(x)}$$

2.2.4 Propose Three-Dimensional Stressed Logit Default Probability Models

TYPE 1:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)}$$

TYPE 2:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)}$$

TYPE 3:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm f_2(x) \pm \sigma_A f_3(x)}$$

TYPE 4:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm f_3(x)}$$

TYPE 5:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm f_2(x) \pm \sigma_A f_3(x)}$$

TYPE 6:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm f_2(x) \pm f_3(x)}$$

TYPE 7:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm \sigma_A f_2(x) \pm f_3(x)}$$

TYPE 8:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{k} \beta_i X_i\right) \pm f_1(x) \pm f_2(x) \pm f_3(x)}$$

TYPE 9:

$$PD_{Stressed} = \frac{1}{1 + \exp\left(\sum_{i=0}^{k} \beta_i X_i\right) \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)}$$

2.2.5 Propose Two-Dimensional Stressed Probit Default Probability Models

TYPE 1:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm \sigma_A f_2(x)$$

TYPE 2:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)\right] \pm f_1(x) \pm \sigma_A f_2(x)$$

TYPE 3:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm f_2(x)$$

TYPE 4:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)\right] \pm f_1(x) \pm f_2(x)$$

TYPE 5:

$$PD_{Stressed} = \phi\left[\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm \sigma_A f_2(x)$$

2.5.6 Propose Three-Dimensional Stressed Probit Default Probability Models

TYPE 1:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)$$

TYPE 2:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)$$

TYPE 3:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm f_2(x) \pm \sigma_A f_3(x)$$

TYPE 4:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm f_3(x)$$

TYPE 5:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm f_1(x) \pm f_2(x) \pm \sigma_A f_3(x)$$

TYPE 6:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm f_2(x) \pm f_3(x)$$

TYPE 7:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm f_1(x) \pm \sigma_A f_2(x) \pm f_3(x)$$

TYPE 8:

$$PD_{Stressed} = \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm f_1(x) \pm f_2(x) \pm f_3(x)$$

TYPE 9:

$$PD_{Stressed} = \phi\left[\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f_1(x) \pm \sigma_A f_2(x) \pm \sigma_A f_3(x)$$

2.2.7 Propose Jameel'S N-Dimensional Stressed Default Probability Theorem

Let $\{W_{JB1}(x),\ W_{JB2}(x),\ W_{JB3}(x),\dots,W_{JBn}(x)\},\ x\ is\ the\ Asset's\ Return$s be a set of *Non-Normal Fat-tailed Probability Distributions satisfies Jameel's Criterion* with RANKING *1st, 2nd, 3rd,…, nth* respectively. Let σ_A be a Geometric Volatility of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters and $\mu_A > 0, Infinitesimal$ be a Geometric Means of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters such that:

$$L_1(x) \coloneqq \exp \mu_A \left(\sum_{i=0}^{k} \beta_i x_i\right) \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x)$$

$$L_2(x) \coloneqq \exp \mu_A \left(\sum_{i=0}^{k} \beta_i x_i\right) \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB3}(x)$$

$$L_3(x) \coloneqq \exp \mu_A \left(\sum_{i=0}^{k} \beta_i x_i\right) \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB(n-10)}(x)$$

$$L_4(x) \coloneqq \exp \mu_A \left(\sum_{i=0}^{k} \beta_i x_i\right) \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB(n-5)}(x) \pm \sigma_A W_{JB(n-4)}(x)$$

$$L_n(t) \coloneqq \exp \mu_A \left(\sum_{i=0}^{k} \beta_i x_i\right) \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \cdots \pm \sigma_A W_{JBn}(x)$$

Such that

$$PD_{Stressed} = \frac{1}{1 + L_i(x)}, i = 1,2,\dots,n$$

And that

$$L_1(x) \coloneqq \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j x_j\right)\right] \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x)$$

$$L_2(x) \coloneqq \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j x_j\right)\right] \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB3}(x)$$

$$L_3(x) \coloneqq \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j x_j\right)\right] \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB(n-10)}(x)$$

$$L_4(x) \coloneqq \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j x_j\right)\right] \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \sigma_A W_{JB(n-5)}(x) \pm \sigma_A W_{JB(n-4)}(x)$$

$$L_n(t) \coloneqq \phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J} \beta_j x_j\right)\right] \pm \sigma_A W_{JB1}(x) \pm \sigma_A W_{JB2}(x) \pm \cdots \pm \sigma_A W_{JBn}(x)$$

Such that

$$PD_{Stressed} = L_i(x), i = 1,2,\dots,n$$

Then we generated a set $\{L_1(x),\ L_2(x),\ L_3(x),\dots,L_n(x)\}$of ARBITRARY COMBINATIONS EXCLUDING FIRST FEW TERMS OF **DIFFERENT DIMENSIONS**. Then $\exists\ L_i(x), i = 1,2,\dots,n$ such that:

(i) $PD_{Stressed} = \frac{1}{1+L_i(x)}$is Optimal reference to LOGIT and CONVERGE TO REAL LIFE DEFAULT PROBABILITIES for each $1,2,\dots,n$.

(ii) $PD_{Stressed} = L_i(x)$is Optimal reference to PROBIT and CONVERGE TO REAL LIFE DEFAULT PROBABILITIES for each $1,2,\dots,n$.

(iii) Or the difference between the **MODEL DEFAULTPROBABILITIES** and **REAL LIFEDEFAULT PROBABILITIES** will be very **NEGLIGIBLE** or even possibly **ZERO** at many points in time t. Note that one can work out for the other proposed model TYPES.

3. Empirical Results

Assume, the data distribution Mean equal 0, Standard Deviation equal 1 for **LOG-LOGISTIC (3P)**$\equiv$ $\boldsymbol{f_1(x)}$, **CAUCHY** $\equiv$ $\boldsymbol{f_2(x)}$ and **BURR(4P)** $\equiv$ $\boldsymbol{f_3(x)}$ **.**While, $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$.**Assume**$\sum_{i=0}^{k} \beta_i X_i = \sum_{j=1}^{J} \beta_j X_j = 0.4673$ and $\beta_0 = 0.0703$.

Then we canPredict Twenty One (21) working days (from 12/1/ 2014 to 12/30/ 2014) Default Probabilities andcompare it with **NORMAL DEFAULT PROBABILITIES**.

The Author performs the PREDICTION Using MICROSOFT EXCEL and obtained the following RESULTS as shown in Tables below:

Note that in Table 1: the notation**T1-2D LOGIT ++** means Type 1 Two-Dimensional Logit***(with respect to Log-Logistic (3P) and Cauchy)*** ++ etc, Table 2: the notation**T1-3D LOGIT +++** means Type 1 Three-Dimensional Logit***(with respect to Log-Logistic (3P), Cauchy and Burr (4P))*** +++ etc, Table 3: the notation**T1-2D PROBIT ++** means Type 1 Two-Dimensional Probit***(with respect to Log-Logistic (3P) and Cauchy)*** ++ etc and Table 4: the notation**T1-3D PROBIT +++** means Type 1 Three-Dimensional Probit***(with respect to Log-Logistic (3P), Cauchy and Burr (4P))*** +++ etc.

Table 1. Propose Two-Dimensional Stressed Logit Default Probability Models TYPE 1

Date	Time	Price	Return	Normal PD	T1-2D LOGIT ++	T1-2D LOGIT -+	T1-2D LOGIT+ -	T1-2D LOGIT- -
12/1/2014	1	111.730003	0.020288572	**0.399106529**	0.487854521	0.487874481	0.505333318	0.505354734
12/2/2014	2	114.019997	-0.002722506	**0.398945237**	0.487860892	0.487861289	0.505347471	0.505347898
12/3/2014	3	113.709999	-0.012655596	**0.399006182**	0.48785788	0.487866879	0.505341474	0.505351129
12/4/2014	4	112.279999	-0.012637372	**0.399005998**	0.48785789	0.487866862	0.505341493	0.505351119
12/5/2014	5	110.870003	-0.037400416	**0.399500708**	0.487828654	0.487916991	0.505287719	0.505382493
12/8/2014	6	106.800003	0.001964352	**0.39894382**	0.487860959	0.487861162	0.505347608	0.505347826
12/9/2014	7	107.010002	-0.02029617	**0.399106652**	0.487852514	0.487876494	0.505331159	0.505356887
12/10/2014	8	104.860001	0.000476741	**0.398942371**	0.487861024	0.487861036	0.505347743	0.505347756
12/11/2014	9	104.910004	-0.024411526	**0.39918009**	0.487848372	0.48788374	0.505323385	0.505361332
12/12/2014	10	102.379997	-0.014957925	**0.39903155**	0.487856564	0.487869268	0.50533891	0.505352542
12/15/2014	11	100.860001	0.008293847	**0.398969724**	0.487859851	0.487863366	0.505345243	0.505349015
12/16/2014	12	101.699997	0.041600454	**0.399633287**	0.487837288	0.487913939	0.505290992	0.505373226
12/17/2014	13	106.019997	0.027995337	**0.39925507**	0.487849257	0.487886021	0.505320938	0.505360383
12/18/2014	14	109.029999	0.035145093	**0.399435349**	0.487843338	0.487899547	0.505306428	0.505366734
12/19/2014	15	112.93	-0.008001474	**0.398967823**	0.487859808	0.487863329	0.505345283	0.505349061
12/22/2014	16	112.029999	0.016993046	**0.399057497**	0.487856363	0.487870567	0.505337517	0.505352758
12/23/2014	17	113.949997	-0.004221236	**0.398949389**	0.487860697	0.48786166	0.505347074	0.505348107
12/24/2014	18	113.470001	-0.001940729	**0.398943783**	0.487860959	0.48786116	0.50534761	0.505347826
12/26/2014	19	113.25	0.000617911	**0.398942433**	0.487861021	0.487861041	0.505347738	0.505347759
12/29/2014	20	113.32	-0.00185487	**0.398943653**	0.487860965	0.487861149	0.505347622	0.505347819

Table 2. Propose Three-Dimensional Stressed Logit Default Probability Models TYPE 1

Normal PD	T1-3D LOGIT+++	T1-3D LOGIT++-	T1-3D LOGIT- -+	T1-3D LOGIT+-+	T1-3D LOGIT-+-	T1-3D LOGIT---	T1-3D LOGIT--+	T1-3D LOGIT+--
0.399106529	0.494713274	0.481183349	0.512718097	0.512696053	0.481202766	0.498199873	0.512718097	0.498179059
0.398945237	0.494563064	0.481337942	0.512542693	0.512542254	0.48133833	0.4983523	0.512542693	0.498351885
0.399006182	0.494492321	0.481399107	0.512473358	0.51246343	0.481407869	0.498424152	0.512473358	0.49841476
0.399005998	0.494492455	0.481398999	0.512473481	0.512463582	0.481407735	0.498424016	0.512473481	0.498414652
0.399500708	0.494294165	0.481530101	0.512324992	0.512227596	0.481616171	0.498625634	0.512324992	0.498533377
0.39894382	0.494595066	0.481307764	0.512576915	0.512576691	0.481307962	0.498319811	0.512576915	0.498319599
0.399106652	0.494434817	0.481443166	0.512423438	0.512396985	0.48146652	0.498482588	0.512423438	0.498457554
0.398942371	0.494584996	0.481317426	0.512565957	0.512565944	0.481317438	0.498330032	0.512565957	0.49833002
0.39918009	0.494402583	0.481465664	0.512397954	0.512358943	0.481500112	0.498515356	0.512397954	0.49847843
0.39903155	0.494475297	0.481412679	0.512457979	0.512443962	0.48142505	0.498441449	0.512457979	0.498428188
0.398969724	0.494637053	0.481265852	0.512624459	0.512620577	0.481269273	0.498277195	0.512624459	0.498273528
0.399633287	0.494840408	0.481029622	0.512892748	0.512808048	0.481104149	0.498071006	0.512892748	0.497991131
0.39925507	0.494760305	0.481128622	0.512780246	0.512739634	0.481164381	0.498152186	0.512780246	0.498113858
0.399435349	0.494802815	0.481076918	0.51283899	0.512776888	0.481131579	0.498109098	0.51283899	0.498050511
0.398967823	0.494525992	0.481370953	0.512505269	0.512501383	0.481374381	0.498389948	0.512505269	0.498386274
0.399057497	0.494692727	0.481206371	0.51269196	0.512676274	0.481220191	0.498220712	0.51269196	0.498205898
0.398949389	0.494552654	0.481347424	0.512531942	0.512530879	0.481348362	0.498362871	0.512531942	0.498361866
0.398943783	0.494568459	0.481332963	0.512548339	0.512548117	0.481333159	0.498346822	0.512548339	0.498346612
0.398942433	0.494585955	0.481316513	0.512566993	0.512566971	0.481316532	0.498329059	0.512566993	0.498329038
0.398943653	0.49456905	0.481332415	0.512548961	0.512548758	0.481332594	0.498346222	0.512548961	0.49834603

Table 3. Propose Two-Dimensional Stressed Probit Default Probability Models TYPE 1

Date	Time	Price	Return	Normal PD	T1-2D LOGIT ++	T1-2D LOGIT -+	T1-2D LOGIT+-	T1-2D LOGIT--
12/1/2014	1	111.730003	0.020288572	**0.399106529**	0.542007891	0.541924029	0.471108377	0.471024515
12/2/2014	2	114.019997	-0.002722506	**0.398945237**	0.541981125	0.541979453	0.471052953	0.471051281
12/3/2014	3	113.709999	-0.012655596	**0.399006182**	0.541993776	0.54195597	0.471076436	0.47103863
12/4/2014	4	112.279999	-0.012637372	**0.399005998**	0.541993737	0.541956042	0.471076364	0.471038669
12/5/2014	5	110.870003	-0.037400416	**0.399500708**	0.542116582	0.541745447	0.471286959	0.470915824
12/8/2014	6	106.800003	0.001964352	**0.39894382**	0.541980841	0.541979989	0.471052417	0.471051564
12/9/2014	7	107.010002	-0.02029617	**0.399106652**	0.542016324	0.541915574	0.471116832	0.471016082
12/10/2014	8	104.860001	0.000476741	**0.398942371**	0.541980569	0.541980519	0.471051887	0.471051837
12/11/2014	9	104.910004	-0.024411526	**0.39918009**	0.542033729	0.541885133	0.471147273	0.470998677
12/12/2014	10	102.379997	-0.014957925	**0.39903155**	0.541999309	0.54194593	0.471086476	0.471033097
12/15/2014	11	100.860001	0.008293847	**0.398969724**	0.541985498	0.541970728	0.471061678	0.471046908
12/16/2014	12	101.699997	0.041600454	**0.399633287**	0.5420803	0.541758268	0.471274138	0.470952106
12/17/2014	13	106.019997	0.027995337	**0.39925507**	0.542030011	0.541875547	0.471156859	0.471002395
12/18/2014	14	109.029999	0.035145093	**0.399435349**	0.542054879	0.541818724	0.471213682	0.470977527
12/19/2014	15	112.93	-0.008001474	**0.398967823**	0.541985679	0.541970885	0.471061521	0.471046727
12/22/2014	16	112.029999	0.016993046	**0.399057497**	0.542000154	0.541940475	0.471091931	0.471032252
12/23/2014	17	113.949997	-0.004221236	**0.398949389**	0.541981944	0.541977897	0.471054509	0.471050462
12/24/2014	18	113.470001	-0.001940729	**0.398943783**	0.541980842	0.541979995	0.471052411	0.471051564
12/26/2014	19	113.25	0.000617911	**0.398942433**	0.541980581	0.541980496	0.47105191	0.471051825
12/29/2014	20	113.32	-0.00185487	**0.398943653**	0.541980817	0.541980044	0.471052362	0.471051589

Table 4. Propose Three-Dimensional Stressed Probit Default Probability Models TYPE 1

Normal PD	T1-3D PROBIT+++	T1-3D PROBIT++-	T1-3D PROBIT--+	T1-3D PROBIT+-+	T1-3D PROBIT -+-	T1-3D PROBIT ---	T1-3D PROBIT --+	T1-3D PROBIT +--
0.399106529	0.513589384	0.577886715	0.442606008	0.44268987	0.570342536	0.499443021	0.442606008	0.499526883
0.398945237	0.514203322	0.577887699	0.443273477	0.443275149	0.569757257	0.498829084	0.443273477	0.498830756
0.399006182	0.51449259	0.577887328		0.44357525	0.569457156	0.498539815	0.443537444	0.498577622
0.399005998	0.514492043	0.577887329	0.443536976	0.443574671	0.569457735	0.498540362	0.443536976	0.498578057
0.399500708	0.515303294	0.57788429	0.444102537	0.444473671	0.568558735	0.497729112	0.444102537	0.498100247
0.39894382	0.514072493	0.577887708	0.443143216	0.443144068	0.569888338	0.498959913	0.443143216	0.498960766

0.399106652	0.514727785	0.577886715	0.443727542	0.443828292	0.569204114	0.498304621	0.443727542	0.498405372
0.398942371	0.514113656	0.577887716	0.443184923	0.443184973	0.569847433	0.49891875	0.443184923	0.498918801
0.39918009	0.514859651	0.577886265	0.443824599	0.443973195	0.569059211	0.498172755	0.443824599	0.498321351
0.39903155	0.514562215	0.577887173	0.443596004	0.443649382	0.569383023	0.498470191	0.443596004	0.49852357
0.398969724	0.513900866	0.57788755	0.442962275	0.442977046	0.57005536	0.49913154	0.442962275	0.499146311
0.399633287	0.513070053	0.577883468	0.44194186	0.442263892	0.570768514	0.499962353	0.44194186	0.500284385
0.39925507	0.513397237	0.577885805	0.442369621	0.442524085	0.570508321	0.499635168	0.442369621	0.499789633
0.399435349	0.513223589	0.577884694	0.442146237	0.442382392	0.570650014	0.499808817	0.442146237	0.500044972
0.398967823	0.5143549	0.577887561	0.443415949	0.443430742	0.569601664	0.498677506	0.443415949	0.4986923
0.399057497	0.513673341	0.577887015	0.442705439	0.442765118	0.570267288	0.499359065	0.442705439	0.499418745
0.398949389	0.514245884	0.577887674	0.443314402	0.443318449	0.569713957	0.498786522	0.443314402	0.498790569
0.398943783	0.514181263	0.577887708	0.443251985	0.443252832	0.569779574	0.498851143	0.443251985	0.498851989
0.398942433	0.514109735	0.577887716	0.443180979	0.443181064	0.569851342	0.498922671	0.443180979	0.498922755
0.398943653	0.514178847	0.577887709	0.443249619	0.443250392	0.569782014	0.498853559	0.443249619	0.498854332

It can be observed, all the ***Shaded Areas in Table 1 to 4*** shows NORMAL DEFAULT PROBABILITIES while the ***Un-shaded Areas***shown the performances of the proposed Dimensional Default Probability Models vis-à-vis NORMAL DEFAULT PROBABILITIES. However, the scope of the research work is to compare the proposed Dimensional Default Probability Models with the Normal Default Probabilities as the Real LifeDefault Probabilities are NOT at the Author's disposal.

The results performances were FASCINATINGLY interesting, impressive, viable, reliable, sophisticated and complaint with IFRS 9 since they ***incorporated the forward-looking information*** satisfiesJameel's Criterion and Geometric average of only positive ***Economic forecasts of the future Macroeconomic scenarios*** $\{\mu_A > 0, Infinitesimal\ \boldsymbol{and}\ (\boldsymbol{\sigma_A \geq 1})\}$ and also minimized the differences betweenModel Default Probabilities and REAL LIFE DEFAULT PROBABILITIES.

4. Discussions

In this paper, the performances of the PROPOSED MODELS with respect to LOG-LOGISTIC (3P), CAUCHY, and BURR (4P) can be improved using the following:

(1) Accurate prediction of economic forecasts of fundamental macroeconomic parameters used in the proposed models

(2) The Author set the Log-Logistic (3P) parameter $\boldsymbol{\xi}$ to be 1 and Burr (4P) parameters $a = 1, k = 1, \gamma = 1, \beta = 1\ and\ \alpha = 2$ thus collapsed to almost Normal. With HIGH VALUES of $\boldsymbol{\xi}$, a, k, γ, β, and α, the proposed Jameel's Stressed Closed Prices will effectively approximates the REAL PRICES.

(3) Jameel's Criterion axiom known as **"Criterion Enhancement Axiom"** : That if we could be able to Runs the Goodness of Fit Tests such as the RANKS of Kolmogorov Smirnov (KS) Test, Anderson-Darling Test, Jarque and Bera (JB) Test, Shapiro Wilk (SW) Test, Cramer-Von Mises Test, Pearson $\left(\chi^2\ Godness\ of\ Fit\right)$ Test, Lilliefors Corrected K-S Test, D'AgostinoSkewness Test, Anscombe-Glynn Kurtosis Test, D'Agostino-Pearson Omnibus are all **UNITY (1)** of the underlying Stock Returns then the proposed Jameel's Stressed Closed Prices will coincide at finitely many points with the REAL PRICES .

(4) $\mu_A$can be **TESTED** as **ARITHMETIC** Means of only positive Arithmetic Means of the Underlying Asset Return and Returns of the future economic forecasts of macroeconomic parameters, otherwise should remains**GEOMETRIC MEANS** as defined and used in the paper.

5. Conclusion

LOGIT and PROBIT Models were first applied to Financial Markets byOhlson (1980) and Zmijewski (1984) to predict bankruptcy and to estimate probability of default respectively.Logit possesses FATTER TAILS than Probit and that makes it more robust to calculate Default probabilities. However, with the advancement in Information and Telecommunication Technology, Natural Disasters, Civil Unrest, Terrorism, Stock Market Crashes and Bubbles, Banks and other Institutions find it very difficult to accurately calculate Default Probabilities of their Borrowers, thus, Logit, Probit and other Default Probability Models needs to be enhanced to be abled accurately Quantify and Predict potential Credit Risk face by those Institutions.

Jamilu (2015) enhanced LOGIT and PROBIT Default Probability Models with the aid of one-dimensional forward-looking information $\{f(x)\}$ satisfies Jameel's Criterion and positive average of economic forecasts of future macroeconomic scenarios $\{\mu_A \, and \, \sigma_A\}$. In this paper, the Author further enhance LOGIT and PROBIT Default Probability Models using Two and Three Dimensional Forward-Looking Information(s) $\{f_1(x), f_2(x)\}$ and $\{f_1(x), f_2(x), f_3(x)\}$ respectively satisfies Jameel's Criterion with LOG-LOGISTIC (3P) $\equiv f_1(x)$, CAUCHY $\equiv f_2(x)$ and BURR(4P) $\equiv f_3(x)$ and positive average of economic forecasts of future macroeconomic scenarios $\{\mu_A \, and \, \sigma_A\}$. The paper tested the performances of only proposed Default Probability Models of TYPE 1 in each class using Twenty One (21) working days (from 12/1/ 2014 to 12/30/ 2014). The results were fascinatingly interesting, impressive, viable and reliable, sophisticated, and complaint with IFRS 9 since they incorporated forward-looking information and Economic forecasts of the future macroeconomic scenarios thereby minimizing the differences between MODELS DEFAULT PROBABILITIES and REAL LIFE DEFAULT PROBABILITIES.

Also, Jameel's n-dimensional Stressed Default Probability Theorem was proposed in the paper, however, the Theorem is expected to sufficiently approximate REAL LIFE Default Probabilities.

In the case of future research direction, one can use the Theorem to find ARBITRARY COMBINATIONS EXCLUDING FIRST FEW TERMS OF DIFFERENT DIMENSIONS $L_j(x)$ among $\{L_1(x), L_2(x), L_3(x), \ldots, L_n(x)\}$ such that $L_j(x) - P_i(x) = 0 \; at \; every \; point, j = 1,2, \ldots, m, i = 1,2, \ldots, k, 0 \le P_i(x) \le 1$where $P_i(x), x \; is \; the \; Asset's \; Return$ is the REAL LIFE Default Probabilities correspond to eachMODEL Default Probabilities $L_i(x)$.

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