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Multiple Contracts with Simple Interest: The Case of SACRE-F

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Abstract

In a previous paper de Faro and Lachtermacher (2023), considering a particular version of the so-called System of Increasing Amortization in Real Terms, SACRE-F, in the case of compound interest, was shown that if a single contract is substituted by multiple contracts, one for each payment of the single contract, the financing institution may derive substantial fiscal gains. In terms of the present value of the corresponding income taxes.

Notwithstanding, the Brazilian Jurisprudence, cf. Jusbrasil (2023), has repeatedly determined that the use of compound interest implies the occurrence of anatocism. The payment of interest on interest. Mari & Aretusi (2019) reported the same kind of determination by the Italian justice.

Therefore, the analysis should also consider the hypothesis of the use of simple interest. SACRE-F, in the case of simple interest has shown that if a single contract is substituted by multiple contracts, one for each payment of the single contract, the financing institution may derive substantial fiscal gains, as in the compound interest case.

Keywords: SACRE-F System of Increasing Amortization in Real Terms in simple interest case, Amortization System in Simple Interest

1. Introduction

In a previous paper de Faro and Lachtermacher (2023), considering a particular version of the so-called System of Increasing Amortization in Real Terms ("*Sistema de Amortizações Crescentes em Termos Reais*"), SACRE-F, it was shown that if a single contract is substituted by multiple contracts, one for each payment of the single contract, the financing institution may derive substantial fiscal gains. In terms of the present value of the corresponding income taxes.

However, the analysis was conducted under the assumption that the transactions were made in terms of compound interest. Whose principles prevail in the financial transactions all over the world.

Notwithstanding, the Brazilian Jurisprudence, cf. Jusbrasil (2023), has repeatedly determined that the use of compound interest implies the occurrence of anatocism. The payment of interest on interest. Mari & Aretusi (2019) reported the same kind of determination by the Italian justice. Therefore, the analysis should also consider the hypothesis of the use of simple interest.

At this point, it is appropriate to single out the conflicting arguments recently presented in Puccini (2023) and in De-Losso et al. (2023). Regarding the occurrence, or not, of anatocism in any system of amortization employing compound interest. A question that is also present in the Italian literature; cf. Marcelli et al. (2019).

In what follows, we will address the case where it is supposed that the contracts have been written in terms of simple interest.

2. Using Simple Interest

Considering a loan F, suppose that it must be repaid by n periodic payments. If simple interest is used, at the periodic interest rate i, the first step is to consider the requirement of the specification of what is called a focal date; cf. Ayres (1963). As, in opposition to the case of compound interest, when the choice of a focal date is irrelevant, distinct focal dates imply in different results in the case of simple interest.

We are going to consider two distinct focal dates. The first one, time zero, which should be considered as the most natural, and is the one established in a Brazilian Law of 1964, cf. De-Losso et al. (2020), implies that the k-th periodic payment, denoted as p_k, must be such that:

$$F = \sum_{k=1}^{n} \frac{p_k}{1 + i \times k} \tag{1}$$

While the second one, time *n*, which has been repeatedly specified in the case of constant payments, as in Nogueira (2013), and in the case of constant amortization, as in Rovina (2009), implies that the corresponding *k*-th periodic payments, now denoted as $\hat{p}_k$, must be such that:

$$F \times \left(1 + i \times n\right) = \sum_{k=1}^{n} \hat{p}_k \times \left\{1 + i \times \left(n - k\right)\right\} \tag{2}$$

3. Capitalized and non-Capitalized Components

In general terms, Forger (2009) established a decomposition of *F* into two components: capitalized, S_0^C, and non-capitalized S_0^N, as:

$$S_0^C = F \times f \tag{3}$$

$$S_0^N = F \times \left(1 - f\right) \tag{4}$$

with

$$0 \le f \le 1 \tag{5}$$

The weigh factor, *f*, being depended on the system of amortization under consideration, as well on the specification of the focal date.

For this purpose, denoting by S_k the state of the debt, or outstanding balance, at time *k*, immediately following the payment of P_k, and by A_k the corresponding amortization component, it is assumed that we have $S_k = S_k^C + S_k^N$, $P_k = P_k^C + P_k^N$ and $A_k = A_k^C + A_k^N$. With the superscripts *C* and *N* identifying the corresponding capitalized and non-capitalized components.

Forger (2009) also postulates that for any amortization schema, for $k = 1, 2, \ldots, n$, the following relations are valid.

$$S_k^C = S_{k-1}^C - A_k^C = S_{k-1}^C - P_k^C \Leftrightarrow A_k^C = P_k^C \tag{6}$$

and

$$S_k^N = S_{k-1}^N - A_k^N = S_{k-1}^N - P_k^N + J_k \Leftrightarrow A_k^N = P_k^N - J_k \tag{7}$$

with J_k denoting the interest component of P_k.

Additionally, regardless of the system of amortization under scrutiny, and independently of *k*, it is supposed that:

$$P_k^C = A_k^C = F \times f / n = P^C = A^C \tag{8}$$

Recursively, making use of relations (6) and (8), it follows that:

$$S_k^C = F \times f \times \left(n - k\right) / n \tag{9}$$

Furthermore, also independently of any system of amortization, it is supposed that the rate *i* of simple interest applies only in the capitalized component of the outstanding balance.

That is, Forger (2009) postulates that:

$$J_k = i \times S_{k-1}^C \tag{10}$$

Therefore, considering relation (9), we can write:

$$J_k = i \times F \times f \times (n-k+1)/n \tag{11}$$

Finally, to assure an amortization system that is financially consistent, it is assumed that the outstanding balance at the end of the term of *n* periods, is null. That is:

$$S_n = S_n^C = S_n^N = 0 \tag{12}$$

4. General Relations using SACRE-F in Simple Interest Schema

In SACRE-F, whether considering the compound interest regime, or adopting the simple interest regime, the total financing term, of *n* periods, is subdivided into equal ℓ subperiods, each with *m* periods. With *n*, ℓ and *m* being positive integers, and such that the relation $n = \ell \times m$ is valid. Since SACRE-F's logic establishes that, at the end of each subperiod, the remaining outstanding balance is divided by the number of periods in the remainder of the financing term; with the value of the payment remaining constant over the *m* subsequent periods.

To identify the period *k* that is considered, the following relationship will be used:

$$k = (s-1) \times m + q \quad for\ k = 1, 2, ..., n \tag{13}$$

where *s*, such that $1 \leq s \leq \ell$, specifies the subperiod where period *k* lies, and *q*, such that $1 \leq q \leq m$, establishes where, within the subperiod, period *k* lies.

Thus, for example, if *m* = 12, the period *k* = 46 is the 10th component, *q* = 10 of the fourth subperiod, $s = 4$. Using this notation, expressions (1) and (2) can be respectively rewritten as:

$$F = \sum_{s=1}^{\ell} \sum_{q=1}^{m} \frac{p_{(s-1)\times m+q}}{\left\{1 + i \times \left[(s-1) \times m + q\right]\right\}} \tag{14}$$

and

$$F \times (1 + i \times \ell \times m) = \sum_{s=1}^{\ell} \sum_{q=1}^{m} \hat{p}_{(s-1)\times m+q} \times \left\{1 + i \times \left[\ell \times m - (s-1) \times m - q\right]\right\} \tag{15}$$

Considering the general case, making use, recursively, of relation (7), and taking account relation (4) we have that:

$$S_k^N = F \times (1-f) + k \times i \times F \times f - \left[i \times P^C \times k \times (k-1)/2\right] - \sum_{j=1}^{k} P_j^N \tag{16}$$

where $P^C = A^C$ is given by relation (8). In other words, we need only to specify the behavior of non-capitalized payments.

Let us now focus attention to the specific case of SACRE-F, which assumes that, in each of the sub-periods, we have constant payments. Therefore, we should assume that, in each sub-period the corresponding non-capitalized components also remain constant.

Thus, creating the notation *N'*, we have that:

$$P_k^N = P_1^{N'}, \text{ para } k = 1, 2, \ldots, m$$
$$P_k^N = P_2^{N'}, \text{ para } k = m+1, m+2, \ldots, 2m$$
$$\vdots$$
$$P_k^N = P_\ell^{N'}, \text{ para } k = n-m+1, n-m+2, \ldots, n = \ell \times m$$

That is, with this notation, we can write:

$$P_{(s-1)\times m+q}^N = P_s^{N'} \tag{17}$$

On the other hand, according to Forger (2010), it is established that, for each of the ℓ subperiod, the sum of the corresponding non-capitalized amortization installments is constant. Therefore:

$$\sum_{k=1}^{m} A_k^N = \sum_{k=m+1}^{2m} A_k^N = \cdots \sum_{k=n-m+1}^{n} A_k^N = A^{N'} \tag{18}$$

Thus, making use, recursively, of relation (7), we can write:

$$S_{s\times m}^N = S_0^N - s \times A^{N'} \tag{19}$$

Therefore, since $S_0^N = F \times (1-f)$, as given by relation (4), and we must have $S_n^N = F \times (1-f) - \ell \times A^{N'}$, it follows that:

$$A^{N'} = F \times (1-f)/\ell \tag{20}$$

Consequently, we can write:

$$S_{s\times m}^N = F \times (1-f) - s \times F \times (1-f)/\ell$$

or

$$S_{s\times m}^N = F \times (1-f) \times (\ell - s)/\ell \tag{21}$$

For determining the expression of $P_{(s-1)\times m+q}^N$, which will be determined using relation (7), it is convenient to rewrite relation (11), making use of relation (13), and remembering that $n = \ell \times m$,in such a way that:

$$J_{(s-1)\times m+q} = i \times F \times f \times \left[(\ell - s + 1) \times m - (q-1) \right]/n \tag{11'}$$

So, bearing in mind that, in each of the ℓ sub-periods, non-capitalized payments are constant, we can write the following expression, (which appears to be tautological, but which makes sense given the development below):

$$P_{(s-1)\times m+q}^N = P_p^{N'} = \sum_{j=1}^{m} P_{(s-1)\times m+j}^N / m$$

or, in view of relation (7) and relation (18)

$$P_p^{N'} = A^{N'}/m + \sum_{j=1}^{m} J_{(s-1)\times m+j}^N / m$$

Thus, after some algebraic manipulations, we have:

$$P_s^{N'} = \left\{ F \times (1-f) + i \times F \times f \left[(\ell - s + 1) \times m - (m-1)/2 \right] \right\}/n \tag{22}$$

Let us now move on to the determination of the non-capitalized amortizations in each period *k*. Using, once again, relation (7), it follows that:

$$A_{(s-1)\times m+q}^N = P_s^{N'} - J_{(s-1)\times m+q}$$

from what follows that

$$A_{(s-1)\times m+q}^N = \left\{ F \times (1-f) + i \times F \times f \times \left[(q-1) - (m-q)/2 \right] \right\}/n \tag{23}$$

Therefore, the relationship that expresses the evolution of the non-capitalized debt balance can be rewritten as:

$$S_{(s-1)\times m+q}^N = F \times \left\{ (1-f) \times \left[n - (s-1) \times m - q \right] + F \times f \times q \times (m-q)/2 \right\}/n \tag{16'}$$

Up to this point, nothing depended on the focal date chosen for the solutions of equations (14) and (15). In other words, it remains to determine the corresponding expressions for the weighting factor *f*.

In what follows, to give a simple numerical example, we are going to consider the case where F=$120,000.00, n = 12 periods, and that the periodic rate of simple interest is fixed at i = 1% per period.

Additionally, considering the peculiar characteristics of SACRE-F, where the number n of periods is subdivided into ℓ subperiods, with m constant payments in each of the subperiods, so that $n = \ell \times m$, we will suppose that ℓ = 4 and m = 3.

4.1- Focal Date at Time n

Given that in his proposition Forger (2010), only addressed the case where the focal date at time n was considered, we will start our analysis with this case.

According to Forger (2010), the value of weigh factor f is given by the following analytical expression:

$$f = \left\{1 + i \times \left[4 \times n^2 - m^2 - 3\right] / \left[6 \times (n+1)\right]\right\}^{-1} \tag{24}$$

Therefore, given that in our simple numerical example we have i = 1%, n = 12 and m = 3 it follows that f = 0,932568149.

In Table 1 we have the evolution of the sequences of payments, of the parcels of interest, denoted as J'_k as well of the evolution of the outstanding debt in terms of both the capitalized and non-capitalized components. Table 1a resumes the totalization of all the components.

Table 1. SACRE-F Single Contract with Focal Date on Time *n*

		f = 0,932568149		i = 1%		n = 12		
k	J'_k	A_k^N	A_k^C	P_k^N	P_k^C	S_k^N	S_k^C	S_k
0						8,091.82	111,908.18	120,000.00
1	1,119.08	581.06	9,325.68	1,700.14	9,325.68	7,510.76	102,582.50	110,093.26
2	1,025.82	674.32	9,325.68	1,700.14	9,325.68	6,836.44	93,256.81	100,093.26
3	932.57	767.58	9,325.68	1,700.14	9,325.68	6,068.87	83,931.13	90,000.00
4	839.31	581.06	9,325.68	1,420.37	9,325.68	5,487.80	74,605.45	80,093.26
5	746.05	674.32	9,325.68	1,420.37	9,325.68	4,813.49	65,279.77	70,093.26
6	652.80	767.58	9,325.68	1,420.37	9,325.68	4,045.91	55,954.09	60,000.00
7	559.54	581.06	9,325.68	1,140.60	9,325.68	3,464.85	46,628.41	50,093.26
8	466.28	674.32	9,325.68	1,140.60	9,325.68	2,790.53	37,302.73	40,093.26
9	373.03	767.58	9,325.68	1,140.60	9,325.68	2,022.96	27,977.04	30,000.00
10	279.77	581.06	9,325.68	860.83	9,325.68	1,441.89	18,651.36	20,093.26
11	186.51	674.32	9,325.68	860.83	9,325.68	767.58	9,325.68	10,093.26
12	93.26	767.58	9,325.68	860.83	9,325.68	0.00	0.00	0.00
∑	7,274.03	8,091.82	111,908.18	15,365.85	111,908.18			

Table 1a. SACRE-F Single Contract with Focal Date on Time *n* – Consolidation

$\ell = 4$	m=3	f = 0,932568149	i = 1%	n = 12		
s	q	k	J'_k	A_k	$\hat{p}_k$	S_k
		0				120,000.00
1	1	1	1,119.08	9,906.74	11,025.82	110,093.26
1	2	2	1,025.82	10,000.00	11,025.82	100,093.26
1	3	3	932.57	10,093.26	11,025.82	90,000.00
2	1	4	839.31	9,906.74	10,746.05	80,093.26
2	2	5	746.05	10,000.00	10,746.05	70,093.26
2	3	6	652.80	10,093.26	10,746.05	60,000.00
3	1	7	559.54	9,906.74	10,466.28	50,093.26
3	2	8	466.28	10,000.00	10,466.28	40,093.26
3	3	9	373.03	10,093.26	10,466.28	30,000.00
4	1	10	279.77	9,906.74	10,186.51	20,093.26
4	2	11	186.51	10,000.00	10,186.51	10,093.26
4	3	12	93.26	10,093.26	10,186.51	0.00
		∑	7,274.03	120,000.00	127,274.03	

4.2 Focal Date at Time 0

In this eventuality, since an analytical solution for the weighting factor f appears to be impractical, it is suggested that use be made of the general methodology presented in Lachtermacher and de Faro (2023). Which can be easily implemented using Excel spreadsheets. Therefore, given that in our simple numerical example we have i = 1%, n = 12 and m = 3 it follows that f = 0.96704380211.

In Table 2 we have the evolution of the sequences of payments, of the parcels of interest, as well of the evolution of the outstanding debt in terms of both the capitalized and non-capitalized components. Table 2a resumes the totalization of all the components.

Table 2. SACRE-F Single Contract with Focal Date on Time 0

		f = 0.96704380211		i = 1%		n = 12		
k	J_k	A_k^N	A_k^C	P_k^N	P_k^C	S_k^N	S_k^C	S_k
0						3,954.74	116,045.26	120,000.00
1	1,160.45	232.86	9,670.44	1,393.31	9,670.44	3,721.89	106,374.82	110,096.70
2	1,063.75	329.56	9,670.44	1,393.31	9,670.44	3,392.32	96,704.38	100,096.70
3	967.04	426.27	9,670.44	1,393.31	9,670.44	2,966.06	87,033.94	90,000.00
4	870.34	232.86	9,670.44	1,103.20	9,670.44	2,733.20	77,363.50	80,096.70
5	773.64	329.56	9,670.44	1,103.20	9,670.44	2,403.64	67,693.07	70,096.70
6	676.93	426.27	9,670.44	1,103.20	9,670.44	1,977.37	58,022.63	60,000.00
7	580.23	232.86	9,670.44	813.08	9,670.44	1,744.51	48,352.19	50,096.70
8	483.52	329.56	9,670.44	813.08	9,670.44	1,414.95	38,681.75	40,096.70
9	386.82	426.27	9,670.44	813.08	9,670.44	988.69	29,011.31	30,000.00
10	290.11	232.86	9,670.44	522.97	9,670.44	755.83	19,340.88	20,096.70
11	193.41	329.56	9,670.44	522.97	9,670.44	426.27	9,670.44	10,096.70
12	96.70	426.27	9,670.44	522.97	9,670.44	0.00	0.00	0.00
∑	7,542.94	3,954.74	116,045.26	11,497.69	116,045.26			

Table 2a. SACRE-F Single Contract with Focal Date on Time 0 – Consolidation

$\ell = 4$	m = 3	f = 0.96704380211	i = 1%	n = 12		
s	q	k	J_k	A_k	P_k	S_k
		0				120,000.00
1	1	1	1,160.45	9,903.30	11,063.75	110,096.70
1	2	2	1,063.75	10,000.00	11,063.75	100,096.70
1	3	3	967.04	10,096.70	11,063.75	90,000.00
2	1	4	870.34	9,903.30	10,773.64	80,096.70
2	2	5	773.64	10,000.00	10,773.64	70,096.70
2	3	6	676.93	10,096.70	10,773.64	60,000.00
3	1	7	580.23	9,903.30	10,483.52	50,096.70
3	2	8	483.52	10,000.00	10,483.52	40,096.70
3	3	9	386.82	10,096.70	10,483.52	30,000.00
4	1	10	290.11	9,903.30	10,193.41	20,096.70
4	2	11	193.41	10,000.00	10,193.41	10,096.70
4	3	12	96.70	10,096.70	10,193.41	0.00
		∑	7,542.94	120,000.00	127,542.94	

5. The Multiple Contracts Alternative

Rather than engaging a single contract, the financial institution has the option of requiring the borrower to adhere to n subcontracts; one for each of the n payments that would be associated with the case of a single contract. With the principal of the k-th subcontract being depended on the present value, at the same interest rate i, of the k-th payment of the single contract.

Considering the two considered focal dates, we will have:

5.1 Focal Date at Time Zero

Adapting to the case of simple interest the proposition in De-Losso et al. (2013), the principal of the k-th individual contract denoted, as F_k, will be equal to the present value, now at the rate i of simple interest, of the k-th payment of the single contract. That is:

$$F_k = p_k / (1 + i \times k) \quad , \quad k = 1, 2, \ldots, n \tag{25}$$

With the parcel of amortization, $\hat{A}_k$, corresponding to the k-th payments p_k, being exactly equal to F_k. Therefore, from an accounting point of view, the parcel of interest associated to the k-th payment p_k, denoted as $\hat{J}_k$, will be:

$$\hat{J}_k = p_k - \hat{A}_k = p_k \times \left[1 - (1 + i \times k)^{-1}\right] \quad , \quad k = 1, 2, \ldots, n \tag{26}$$

From a strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the subcontracts, the total of interest payments is the same in both cases. However, in terms of present values, and depending on the financial institution opportunity cost, it is possible that the financial institution will be better off if it adopts the multiple contracts option.

In Table 3, considering the case of our numerical example, we show the corresponding components in the case of 12 multiple contracts.

Table 3. Multiple Contracts with Focal date at Time 0

f=0.967043802	i = 1%		n = 12		Multiple Contracts		
k	J_k	A_k	P_k	S_k	$F_k = \hat{A}_k$	$\hat{J}_k$	$d_k = J_k - \hat{J}_k$
0				120,000.00			
1	1,160.45	9,903.30	11,063.75	110,096.70	10,954.21	109.54	1,050.91
2	1,063.75	10,000.00	11,063.75	100,096.70	10,846.81	216.94	846.81
3	967.04	10,096.70	11,063.75	90,000.00	10,741.50	322.25	644.80
4	870.34	9,903.30	10,773.64	80,096.70	10,359.26	414.37	455.97
5	773.64	10,000.00	10,773.64	70,096.70	10,260.60	513.03	260.60
6	676.93	10,096.70	10,773.64	60,000.00	10,163.81	609.83	67.10
7	580.23	9,903.30	10,483.52	50,096.70	9,797.68	685.84	**-105.61**
8	483.52	10,000.00	10,483.52	40,096.70	9,706.96	776.56	**-293.04**
9	386.82	10,096.70	10,483.52	30,000.00	9,617.91	865.61	**-478.79**
10	290.11	9,903.30	10,193.41	20,096.70	9,266.74	926.67	**-636.56**
11	193.41	10,000.00	10,193.41	10,096.70	9,183.25	1,010.16	**-816.75**
12	96.70	10,096.70	10,193.41	0.00	9,101.26	1,092.15	**-995.45**
$\sum$	7,542.94	120,000.00	127,542.94		120,000.00	7,542.94	0.00

Strictly from an accounting point of view, there is no gain if a single contract is substituted by multiple contracts since the sums of the corresponding parcels of interest are the same. Hence,

$$\sum_{k=1}^{n} J_k = \sum_{k=1}^{n} \hat{J}_k = 7{,}542.94$$

Yet, depending on the opportunity cost of the financial institution, which will be denoted as ρ the financial institution may derive substantial financial gains in terms of income tax deductions.

In other words, it is possible that:

$$V_1(\rho) = \sum_{k=1}^{n} J_k \times (1+\rho)^{-k} > V_2(\rho) = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k}$$

where the interest rate ρ is supposed to be relative to the same period of the interest rate i.

Moreover, as the sequence of differences, $d_k = J_k - \hat{J}_k$, has only one change of sign, thus characterizing what is termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this case is null, it follows that $\Delta = V_1(\rho) - V_2(\rho) > 0$ for $\rho > 0$.

Tables 4,5,6 and 7, respectively considering the cases where i = 0.5%, 1%, 1.5% and 2% per period, we have the values of the fiscal gain

$$\delta = \left[V_1\left(\rho_2\right)/V_2\left(\rho_2\right) - 1\right] \times 100 \quad (27)$$

where ρ_a is the annual value of the opportunity costs ρ and n_a is the duration of the term contract in years.

Table 4. SACREF-JS - Single x Multiple – Focal Date on Time 0, i = 0.5%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.5524	15.2394	23.0347	30.9133	38.8520	46.8293
10	14.4908	29.9253	46.1227	62.8966	80.0673	97.4700
15	20.8277	43.6560	67.9230	93.0627	118.5751	144.0614
20	26.6425	56.2917	87.7150	119.7999	151.7235	182.9687
25	31.9898	67.7565	105.1564	142.5562	179.0133	214.1121
30	36.9117	78.0408	120.2296	161.5373	201.1602	238.9229

Table 5. SACREF-JS - Single x Multiple – Focal Date on Time 0, i = 1.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.9639	13.9982	21.0794	28.1859	35.2984	42.3995
10	12.6301	25.7917	39.3224	53.0703	66.9007	80.7002
15	17.3911	35.7604	54.6514	73.6714	92.5194	110.9871
20	21.4885	44.2182	67.3098	90.1207	112.2658	133.5599
25	25.0656	51.3967	77.6503	103.0469	127.2683	150.2613
30	28.2179	57.4916	86.0715	113.2116	138.7785	162.8813

Table 6. SACREF-JS - Single x Multiple – Focal Date on Time 0, i = 1.5%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.4921	13.0101	19.5334	26.0437	32.5250	38.9634
10	11.3152	22.9270	34.6964	46.5013	58.2404	69.8335
15	15.1634	30.8038	46.5600	62.1517	77.3856	92.1450
20	18.3577	37.1924	55.8650	73.9724	91.3158	107.8329
25	21.0707	42.4322	63.1856	82.9119	101.5105	119.0330
30	23.4096	46.7648	68.9867	89.7614	109.1477	127.3135

Table 7. SACREF-JS - Single x Multiple – Focal Date on Time 0, i = 2.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.1018	12.1972	18.2685	24.3002	30.2793	36.1945
10	10.3188	20.7866	31.2851	41.7161	52.0021	62.0853
15	13.5650	27.3225	40.9839	54.3431	67.2719	79.7037
20	16.1965	32.4713	48.3453	63.5534	77.9931	91.6592
25	18.3928	36.6056	54.0066	70.3626	85.6711	100.0231
30	20.2608	39.9705	58.4223	75.5041	91.3481	106.1347

In all cases, the fiscal gain is highly significant.

5.2 Focal Date at Time n

On the other hand, if the focal date is at time *n*, it is necessary to make a further adaptation. To assure that the sums of the principals of the subcontracts is equal to the value *F* of the single contract, it is necessary that, as shown in Lachtermacher and de Faro (2023):

$$\hat{F}_k = \hat{p}_k \times \{1 + i \times (n-k)\} / (1 + i \times n) \quad , \ k = 1, 2, \ldots, n \tag{28}$$

Ergo, the parcel of amortization associated with the *k*-th payment is exactly equal to the value of the corresponding principal. On the other hand, the parcel of interest, denoted as $\hat{J}'_k$, will be equal to:

$$\hat{J}_k^{'} = \hat{p}_k \times \{1 - [1 + i \times (n-k)] / (1 + i \times n)\} \quad , \ k = 1, 2, \ldots, n \tag{29}$$

In Table 8, still considering the case of our numerical example, it is shown the corresponding components for the case of the adoption of multiple contracts.

Table 8. Multiple Contracts with Focal Date at Time *n*

f = 0,932568149	*i* = 1%		*n* = 12	Multiple Contracts			
k	J'_k	A_k	$\hat{P}_k$	S_k	$F_k = \hat{A}_k$	$\hat{J}'_k$	$d'_k = J'_k - \hat{J}'_k$
0				120,000.00			
1	1,119.08	9,906.74	11,025.82	110,093.26	10,927.38	98.44	1,020.64
2	1,025.82	10,000.00	11,025.82	100,093.26	10,828.94	196.89	828.94
3	932.57	10,093.26	11,025.82	90,000.00	10,730.49	295.33	637.23
4	839.31	9,906.74	10,746.05	80,093.26	10,362.27	383.79	455.52
5	746.05	10,000.00	10,746.05	70,093.26	10,266.32	479.73	266.32
6	652.80	10,093.26	10,746.05	60,000.00	10,170.37	575.68	77.12
7	559.54	9,906.74	10,466.28	50,093.26	9,812.14	654.14	**-94.60**
8	466.28	10,000.00	10,466.28	40,093.26	9,718.69	747.59	**-281.31**
9	373.03	10,093.26	10,466.28	30,000.00	9,625.24	841.04	**-468.01**
10	279.77	9,906.74	10,186.51	20,093.26	9,277.00	909.51	**-629.74**
11	186.51	10,000.00	10,186.51	10,093.26	9,186.05	1,000.46	**-813.95**
12	93.26	10,093.26	10,186.51	0.00	9,095.10	1,091.41	**-998.16**
∑	7,274.03	120,000.00	127,274.03		120,000.00	7,274.03	**0.00**

Once more, from a strict point of view, there is no gain of a single contract is substituted by multiple contracts. Since the sums of the corresponding parcels of interest is the same. Yet, similarly to the case of focal date at time 0, one should consider the opportunity cost of the financing institution providing the loan.

That is, analogously to the case of focal date at time zero, we will have:

$$V_3(\rho) = \sum_{k=1}^{n} J_k^{'} \times (1+\rho)^{-k} > V_4(\rho) = \sum_{k=1}^{n} \hat{J}_k^{'} \times (1+\rho)^{-k}$$

Tables 9, 10, 11 and 12, shows the values of the corresponding fiscal gain.

Table 9. SACREF-JS - Single x Multiple – Focal Date on Time *n*, *i* = 0.5%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.9853	16.1589	24.4929	32.9607	41.5368	50.1972
10	16.2501	33.9292	52.8653	72.8601	93.7028	115.1843
15	24.8150	53.2480	84.7593	118.6565	154.2054	190.7215
20	33.7425	74.1275	119.8136	169.0797	220.2617	272.0217
25	43.0618	96.4893	157.4709	222.6517	289.2379	355.3882
30	52.7846	120.1972	197.1069	278.0651	359.3003	438.8418

Table 10. SACREF-JS - Single x Multiple – Focal Date on Time *n*, *i* = 1.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.7768	15.7161	23.7907	31.9749	40.2443	48.5761
10	15.7052	32.6893	50.7776	69.7753	89.4814	109.7008
15	23.9400	51.1359	81.0394	112.9847	146.2908	180.3399
20	32.5568	71.1187	114.3502	160.6340	208.4607	256.6478
25	41.5823	92.5778	150.2436	211.4771	273.7753	335.5247
30	51.0243	115.3904	188.1681	264.3647	340.6165	415.1972

Table 11. SACREF-JS - Single x Multiple – Focal Date on Time *n*, *i* = 1.5%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.6346	15.4150	23.3148	31.3088	39.3734	47.4869
10	15.3961	31.9911	49.6100	68.0615	87.1507	106.6910
15	23.4955	50.0738	79.1868	110.1850	142.4150	175.2918
20	31.9971	69.7161	111.8321	156.7794	203.1188	249.7356
25	40.9199	90.8508	147.0909	206.6497	267.1457	327.0575
30	50.2669	113.3527	184.4243	258.6781	332.9120	405.4929

Table 12. SACREF-JS - Single x Multiple – Focal Date on Time *n*, *i* = 2.0%p.m.

	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.5314	15.1971	22.9710	30.8285	38.7468	46.7047
10	15.1970	31.5432	48.8641	66.9708	85.6730	104.7891
15	23.2265	49.4345	78.0775	108.5166	140.1150	172.3070
20	31.6712	68.9046	110.3833	154.5723	200.0726	245.8067
25	40.5441	89.8779	145.3248	203.9578	263.4619	322.3653
30	49.8453	112.2265	182.3664	255.5651	328.7067	400.2074

Similarly to the case of focal date at time zero, we will also have very significant fiscal gains if a single contract is substituted by multiple contracts.

6. Conclusion

Analogously to the cases of the adoption of the constant payments scheme or the constant amortization procedure, if the SACRE-F is chosen, the financial institution providing the loan, should always prefer the multiple contracts version. Even if simple interest is the prevailing regime.

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