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Multiple Contracts with Simple Interest: The Case of Constant Payments

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Abstract

Repeatedly, the Brazilian Judicial System has determined that home-financing contracts written in terms of compound interest, particularly in the case of constant payments, should be substituted by contracts specifying simple interest. This has resulted in the adoption of a procedure known as the "Gauss' Method". It is shown that the implementation of a multiple contracts' version may imply substantial fiscal gains, depending on the financing institution opportunity cost.

Keywords: Amortization Systems, Multiple Contracts Scheme with simple interest capitalization, Constant Payments Amortization System

1. Introduction

Motivated by the concept of anatocism, which consists in applying interest upon interest, the Brazilian Judicial System, cf. Jusbrasil (2023), has repeatedly determined that financial contracts written in terms of compound interest should be substituted by contracts making use of simple interest.

Specifically, for the case of the constant payments scheme, which in Brazil is usually named as "Tabela Price," this has resulted in the adoption of what has been denominated as the "Gauss' Method;" cf. Antonick and Assunção (2006) and Nogueira (2013) – namely, a terminology that inappropriately associates the name of the great German mathematician, Johann Carl Friedrich Gauss, to such a procedure.

Focusing attention on the so called "Gauss' Method," our purpose here is to show that the financial institution granting the loan will be better off if a single contract is substituted by multiple contracts.

Subsidiarily, taking into consideration the work of Forger (2009), we will also address its multiple contracts variant.

2. Basic Concepts

Denoting by F the value that is being financed, consider a single contract with n constant periodic payments, and denote by i the periodic interest rate that is being charged.

If i is of compound interest, it is well known, cf. de Faro and Lachtermacher (2012, p.241), that the value of the constant payment, denoted by P, is:

$$P = F \times i / \left[1-(1+i)^{-n}\right] \tag{1}$$

On the other hand, if the rate i is of simple interest, and the so-called focal date, cf. Ayres (1963), is taken to be the end period of the contract, it follows that the value of the constant payment, now denoted as p, must be such that:

$$F \times (1+n\times i) = p \times \sum_{k=1}^{n}\left[1+i(n-k)\right] \tag{2}$$

Therefore, making use of the sum of the n first natural numbers, which erroneously, is attributed to Gauss, it follows that:

$$p = 2 \times F \times (1 + n \times i) / \{n \times [2 + i \times (n-1)]\} \tag{2'}$$

with the value of p, as given by (2'), being denoted as given by the "Gauss' Method".

As shown in de Faro (2013), we have $p < P$, if $n \geq 2$. Therefore, the debtor is benefited if a single contract, originally written in terms of compound interest, is substituted by one where the same interest rate i is now stipulated to be of simple interest.

Before proceeding, it should be noted, as pointed out by De-Losso et al. (2020), that the specification of the end term of the contract as the focal date, violates a Brazilian law of 1964, which stipulates that the focal date must be the beginning of the contract. This point will be further addressed in section 6.

Notwithstanding, although the "Gauss' Method" is plagued by several financial deficiencies, as discussed in de Faro (2016) and De-Losso et al. (2020), it is still being judicially supported.

3. A Simple Numerical Example

Fixing at 1% the periodic interest rate i of simple interest, consider a loan of 10,000 units of capital with a single contract specifying 12 periodic payments in accordance with the so called "Gauss' Method."

From formula (2'), it follows that the 12 periodic payments will be constant and equal to 884.68 units of capital. Conversely, using formula (1), we would have P = 888.49 units of capital. Clearly, the debtor will experience a windfall gain.

At this point, following Nogueira (2013), who is one of the main proponents of the so called "Gauss' Method," to determine the evolution of the outstanding debt, as well as of the parcels of interest, it is necessary to make use of what is named as "weight index," given by:

$$I = 2F \times i / \{n \times [2 + i \times (n-1)]\} \tag{3}$$

which, in this case, is equal to 7.89889415.

Denoting by S_k the outstanding debt at time k, by J_k the parcel of interest, also at time k, and by A_k the corresponding parcel of amortization, we will have:

$$S_k = F \times \{1 - k \times [2 + i \times (k-1)] / \{n \times [2 + i \times (n-1)]\}\} \tag{4}$$

$$J_k = (n - k + 1) \times I \tag{5}$$

$$A_k = p - J_k \quad \text{for } k = 1, 2, \ldots, n \text{ and } S_0 = F \tag{6}$$

Table 1 presents the evolution of the debt according to the "Gauss' Method".

Table 1. Evolution of the Debt According to the "Gauss' Method"

k	p_k	A_k	J_k	S_k
0	-	-	-	10,000.00
1	884.68	789.89	94.79	9,210.11
2	884.68	797,79	86.89	8,412.32
3	884.68	805.69	78.99	7,606.64
4	884.68	813.59	71.09	6,793.05
5	884.68	821.48	63.19	5,971.56
6	884.68	829.38	55.29	5,142.18

7	884.68	837.28	47.39	4,304.90
8	884.68	845.18	39.49	3,459.72
9	884.68	853.08	31.60	2,606.64
10	884.68	860.98	23.70	1,745.66
11	884.68	868.88	15.80	876.78
12	884.68	876.78	7.90	0.00
$\sum$	10,616.11	10,000.00	616.11	-

Before proceeding, it is imperative to point out that the determination of the outstanding debt at time k, S_k, as given by formula (4), does not agree with the results that would be derived by the well-established concepts of either the retrospective method or by the prospective method, which, following Kellison (1991), states that:

a) According with the prospective method:

The outstanding loan balance at any point in time is equal to the present value at that date of the remaining payments.

For instance, at time 10, just after the 10th payment, as we are using simple interest, we would have:

$$S_{10} = \frac{884.68}{1+0.01} + \frac{884.68}{1+2\times 0.01} = 1,743.25 \quad \text{units of capital.}$$

while formula (4), as shown in Table 1, would imply the value of 1,745.66 units of capital.

b) According with the retrospective method:

The outstanding loan balance at any point in time is equal to the original amount of the loan accumulated to that date less the accumulated value at that date of all payments previously made.

Thus, for instance, the outstanding loan balance just after the third payment, would have to be:

$$S_3 = 10,000\times(1+3\times 0.01) - 884.68\times(1+2\times 0.01+1+1\times 0.01+1) = 7,619.42 \text{ units of capital.}$$

On the other hand, the results presented in Table 1 would imply that S_3 = 7,606.64 units of capital.

To remedy this incongruence, which may result in judicial arguments, de Faro and Lachtermacher (2023) extended the work of Forger (2009) providing a financially consistent procedure. This point will be further addressed in section 6.

However, given that the parcels of interest are not affected, we will proceed with the analysis accordingly.

4. Implementing Multiple Contracts

Rather than engaging a single contract, the financial institution has the option of requiring the borrower to adhere to n subcontracts; one for each of the n payments that would be associated with the case of a single contract.

In the case where the interest rate i is of compound interest, we know, cf. De Losso et al. (2013), de Faro (2022), and de Faro and Lachtermacher (2023a, 2023b) that the principal of the k-th subcontract is the present value, at the compound interest rate i, of the k-th payment of the original single contract.

However, in the present situation, where the interest rate i is of simple interest, and where the focal date is being considered the end term of the contract, an adaptation is thus necessary.

The principal of the k-th subcontract, denoted as F_k, is now defined to be:

$$F_k = p \times [1 + i \times (n-k)]\ (1 + i \times n),\ \text{for } k = 1,2,\dots,n \tag{7}$$

With this proviso, we are assured that the contractual debt F is fully amortized.

As for the k-th parcel of amortization, similarly to the case of compound interest, we also have:

$$\hat{A}_k = F_k, \text{for } k = 1,2,\dots,n \tag{8}$$

On the other hand, regarding the k-th parcel of interest, which will be denoted as $\hat{J}_k$, and is equal to the difference $p - \hat{A}_k$, we will have:

$$\hat{J}_k = p \times i \times k\ (1 + i \times n),\ \text{for } k = 1,2,\dots,n \tag{9}$$

Therefore, considering our simple numerical example of section 3, Table 2 presents the sequence of the 12 constant payments, which is the same in the case of a single contract, as well in the case of the 12 individual subcontracts. Additionally, Table 2 also presents the sequences of payments, the sequences J_k and $\hat{J}_k$, as well as the sequence of differences $d = J_k - \hat{J}_k$.

As previously pointed out, it should be noted that the original debt of 10,000 units of capital is fully amortized, since:

$$\sum_{k=1}^{n} F_k = \sum_{k=1}^{n} \hat{A}_k = F \tag{10}$$

and, in this case with n = 12.

Table 2. The Sequences of the Parcels of Interest and their Differences

k	p_k	J_k	$\hat{J}_k$	d_k
0	-	-	-	-
1	884.68	94.79	7.90	86.89
2	884.68	86.89	15.80	71.09
3	884.68	78.99	23.70	55.29
4	884.68	71.09	31.60	39.49
5	884.68	63.19	39.49	23.70
6	884.68	55.29	47.39	7.90
7	884.68	47.39	55.29	-7.90
8	884.68	39.49	63.19	-23.70
9	884.68	31.60	71.09	-39.49
10	884.68	23.70	78.99	-55.29
11	884.68	15.80	86.89	-71.09
12	884.68	7.90	94.79	-86.89
$\sum$	10,616.11	616.11	616.11	0.00

Strictly from an accounting point of view, there is no gain for the financial institution granting the loan if a single contract is substituted by multiple contracts, since the sum of the corresponding parcels of interest is the same. That is:

$$\sum_{k=1}^{12} J_k = \sum_{k=1}^{12} \hat{J}_k = 616.11 \text{ units of capital.}$$

Yet, depending on the opportunity cost of the financial institution, which will be denoted as ρ, and is usually of compound interest, and which is supposed to be relative to the same period of the simple interest rate i that is being charged, the financial institution may derive substantial gains in terms of tax deductions.

In other words, it is possible that:

$$V_1(\rho) = \sum_{k=1}^{n} J_k \times (1+\rho)^{-k} > V_2(\rho) = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k} \tag{11}$$

where $V_1(\rho)$ denotes the present value, at the rate ρ, of the sequence of the parcels of interest in the case of a single contract, and $V_2(\rho)$ denotes the corresponding present value in the case of the adoption of multiple contracts. Moreover, at least in the case of our simple numerical example, as the sequence d_k of differences has only one change of sign, thus characterizing what is termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this particular case is null, it follows that $\Delta = V_1(\rho) - V_2(\rho) > 0$ if $\rho > 0$.

Figure 1 outlines the evolution of Δ, for $0 \le \rho \le 5\%$ per period, for F = 10.000 units of capital and n = 12. Additionally, we also have the evolution of Δ when the simple interest rate i is equal to 0.5%, 1%, 1.5%, 2%, 2.5%, and 3% per period.

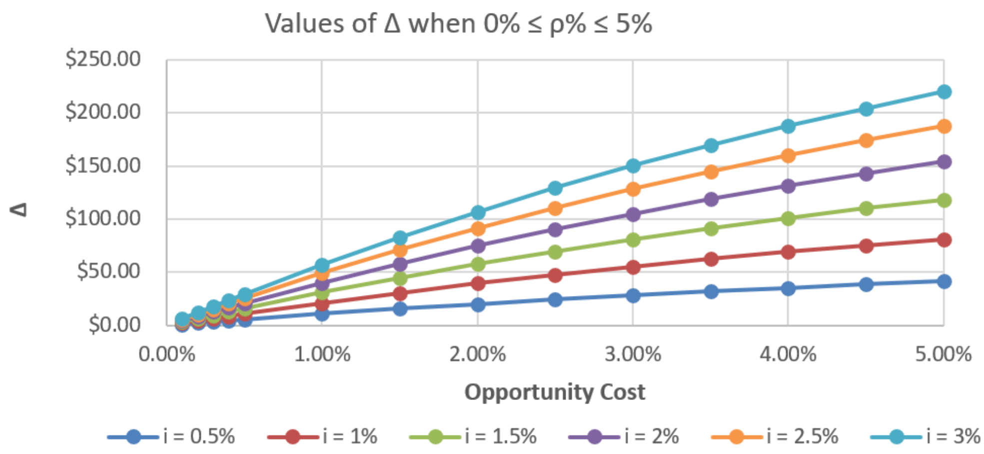

Figure 1. Values of Δ – F =10,000 units of capital and n =12

As indicated, the value of Δ increases both regarding i and ρ.

5. General Analysis

In the previous section, focusing attention on our simple example, with only 12 periods, it was verified that the sequence of differences of the interest payments present just one change of sign, thereby assuring us of the uniqueness of the corresponding internal rate of return, which is known to be null.

Furthermore, this inference appears to always be true, as supported by the evidence provided in Figure 2, which presents the evolution of the sequence d_k for the case where $F = 1{,}200{,}000$ units of capital of a single contract with 180 periods, and with the simple interest rate i being as great as 3% per period.

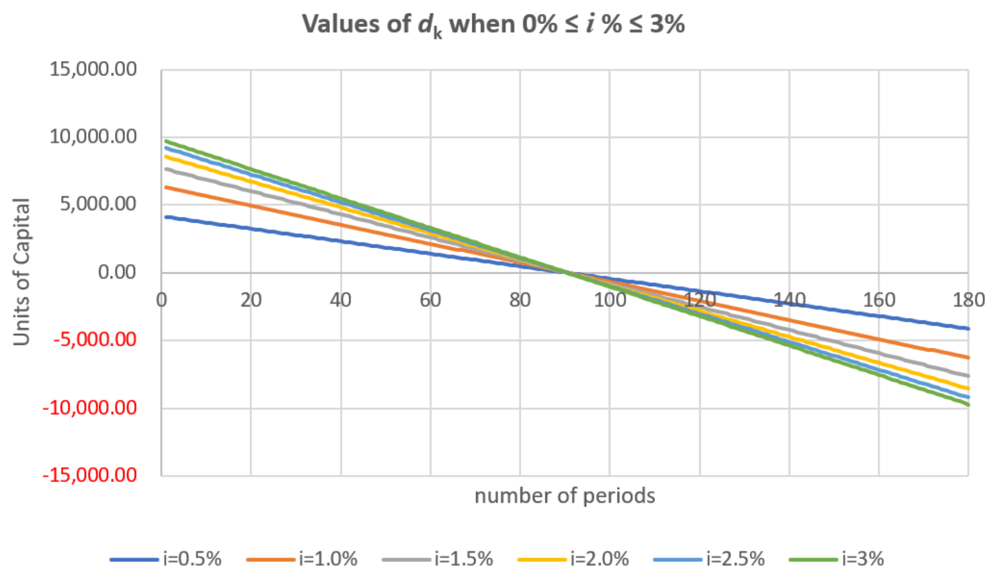

Figure 2. Values of d_k – F =1,200,000 units of capital and n =180

Consequently, it can be inferred that the financing institution is always better off if a single contract is substituted by multiple contracts, one for each one of the n payments of the original contract.

Taking into account that in Brazil the monthly interest rates charged do not exceed 2% per month, in real terms, we are going to analyze the behavior of the percentage increase of the fiscal gain $\delta = [V_1(\rho_a)/V_2(\rho_a) - 1] \times 100$, for some values of the corresponding annual opportunity cost ρ_a, with each contract with a term of n_a years. This is depicted in Tables 3 and 4.

Table 3. Fiscal gain δ – the end term of the contract as the focal date – i = 0.5%p.m.

	ρ_a (%)					
n_a	5%	10%	15%	20%	25%	30%
5	8.3206	16.8744	25.6332	34.5691	43.6552	52.8660
10	17.4672	36.7401	57.6683	80.0576	103.6862	128.3192
15	27.3318	59.4887	96.0380	136.2741	179.3400	224.3538
20	37.9374	85.1995	140.6702	202.3963	268.1493	335.9098
25	49.3026	113.8649	191.0998	276.7112	366.4184	457.0043
30	61.4403	145.3882	246.5568	357.1327	470.6001	583.0412

Table 4. Fiscal gain δ – the end term of the contract as the focal date – i = 1.0%p.m.

	ρ_a (%)					
n_a	5%	10%	15%	20%	25%	30%
5	8.3206	16.8744	25.6332	34.5691	43.6552	52.8660
10	17.4672	36.7401	57.6683	80.0576	103.6862	128.3192
15	27.3318	59.4887	96.0380	136.2741	179.3400	224.3538
20	37.9374	85.1995	140.6702	202.3963	268.1493	335.9098
25	49.3026	113.8649	191.0998	276.7112	366.4184	457.0043
30	61.4403	145.3882	246.5568	357.1327	470.6001	583.0412

As can be seen, while the values of δ increase with the opportunity cost of the financing institution, they are the same for both the case where i =0.5% p.m. and where i =1.0% p.m.

This behavior, which is also present for the other values of i, is due to the peculiar way that was used for the formulation of $\hat{J}_k$. This can look like something incoherent since it was expected that the values should change depending on the interest rate. (A tedious and rather lengthy proof can be provided by the authors in Appendices A).

6. An Alternative

As pointed out in section 2, the so called "Gauss' Method", even though supported by several judicial sentences, violates a still prevailing Brazilian law of 1964.

Hence, it seems appropriate to also consider the case where the focal date coincides with the date of the contract, which is the date stipulated in the 1964 law.

In this situation, the value of the constant payment, now denoted as p', is given by the solution of the following equation:

$$F = \sum_{k=1}^{n} \frac{p'}{1+i\times k} \tag{12}$$

While an analytical solution of the above equation is not practical even in the case where the number n of payments is not very large, the value of p' may be easily determined using a Excel spreadsheet.

However, as previously analyzed in de Faro (2014), we still have the question of how to proceed to establish the evolution of the outstanding debt.

Precisely at this juncture, we can resort to take advantage of the work of Forger (2009), which makes use of the concepts of capitalized and non-capitalized components.

Directing the interested reader to de Faro and Lachtermacher (2023), which presents a detailed discussion of the work of Forger (2009) for the special case of constant payments, we need only to specify the sequence of interest payments.

Following the afore-mentioned reference, the parcel of interest at time k, now denoted as J'_k, is:

$$J'_k = F \times f \times i \times (n-k+1)\ n,\ \text{for } k = 1,2,\dots,n \tag{13}$$

where f is a weigh factor which decomposes the principal F in a capitalized component, F^C, and in a non-capitalized component, F^N; $0 \le f \le 1$.

The value of f, in this case, depends not only on the focal date, but also on the values of F, n, and i.

For our case, where the focal date is the date of the beginning of the contract, time zero, in general, there is no analytical solution. It is necessary to make use of an algorithm as the one proposed in Lachtermacher and de Faro (2022).

Considering the simple numerical example of section 3, considering that the value of the weigh factor can be determined to be $f = 0.982771415$, the value of the constant payment is $p^{'} = 886.57$, as given by the solution of equation (12). Table 5 presents the value of $p^{'}$, and the sequence of the parcels of interest. We also present the evolution the total debt $S_k^{'}$, as given in de Faro and Lachtermacher (2022).

Table 5. Evolution of the Payments and of the Parcels of Interest

k	$p^{'}$	J'_k	S'_k
0	-	-	10,000.00
1	886.57	98.28	9,211.71
2	886.57	90,09	8,415.23
3	886.57	81.90	7,610.56
4	886.57	73.71	6,797.70
5	886.57	65.52	5,976.65
6	886.57	57.33	5,147.42
7	886.57	49.14	4,309.99
8	886.57	40.95	3,464.37
9	886.57	32.76	2,610.56
10	886.57	24.57	1,748.56
11	886.57	16.38	878.38
12	886.57	8.19	0.00
$\sum$	10,638,80	638.80	-

Before proceeding, it should be noted that, as also shown in de Faro and Lachtermacher (2023), the Forger (2009) procedure also satisfies the concept of financial consistency as proposed in de Faro (2014). That is, the determination of the outstanding debt at any point of time can be achieved by any of the classical methods.

7. Multiple Contracts in the Case of Focal Date at Time Zero

In this case, analogously to the case where the interest rate *i* is of compound interest, the principal of the *k*-th subcontract, now denoted as $\hat{F}_k^{'}$, is taken to be equal to the present value, now at the rate *i* of simple interest, of the *k*-th payment of the single contract.

That is:

$$\hat{F}_k^{'} = \frac{p^{'}}{1+i\times k} \quad , \quad \text{for } k = 1,2 \dots, n \tag{14}$$

with the corresponding parcel of amortization, now denoted as $\hat{A}_k^{'}$, being exactly equal to the principal of the subcontract. That is:

$$\hat{A}_k^{'} = \hat{F}_k^{'} \quad , \quad \text{for } k = 1,2, \dots, n \tag{15}$$

As for the corresponding parcel of interest, denoted as $\hat{J}_k^{'}$, since $\hat{J}_k^{'} = p' - \hat{A}_k^{'}$, we will have:

$$\hat{J}_k^{'} = \frac{p^{'}\times i\times k}{1+i\times k} \quad , \quad \text{for } k = 1,2 \dots, n \tag{16}$$

In Table 6, still considering our simple numerical example, we show the value of the constant payment p', the sequence of the interest payments $\hat{J}'_k$, as well as the sequence J'_k (single contract), and the sequence of the values of the differences $d'_k = J'_k - \hat{J}'_k$.

Table 6. Multiple Contracts versus Single Contract with Focal Date at Time 0

k	p'	J_k'	$\hat{J}_k'$	d_k'
1	886.57	98.28	8.78	89.50
2	886.57	90.09	17.38	72.70
3	886.57	81.90	25.82	56.08
4	886.57	73.71	34.10	39.61
5	886.57	65.52	42.22	23.30
6	886.57	57.33	50.18	7.15
7	886.57	49.14	58.00	-8.86
8	886.57	40.95	65.67	-24.72
9	886.57	32.76	73.20	-40.44
10	886.57	24.57	80.60	-56.03
11	886.57	16.38	87.86	-71.48
12	886.57	8.19	94.99	-86.80
$\sum$	10,638.80	638.80	638.80	0.00

Comparatively to the case where the focal date is the end period of the contract, we have the same amount for the total of the interest payments.

However, the financing institution may likewise derive substantial gains in terms of tax deductions.

This is because, denoting by $V_1'(\rho)$ the present value at the interest rate ρ in the case of a single contract, and by $V_2'(\rho)$ the corresponding present value in the case of multiple contracts, it is possible to have:

$$V_1'(\rho) = \sum_{k=1}^{n} J_k' \times (1+\rho)^{-k} > V_2'(\rho) = \sum_{k=1}^{n} \hat{J}_k' \times (1+\rho)^{-k} \quad (17)$$

Moreover, in this case of our simple numerical example, and in several other cases with different values of *i*, *n* and *F* tested, the sequence of differences d_k' also characterizes a conventional project whose internal rate of return is unique, and which in this particular case is null, it follows that $\Delta' = V'_1(\rho) - V'_2(\rho) > 0$ if $\rho > 0$.

Figure 3 outlines the evolution of Δ', not only when $i = 1\%$ per period, but also when *i* assumes the values of 0.5%, 1.5%, 2% and 3%, and when $0 \le \rho \le 5\%$ per period.

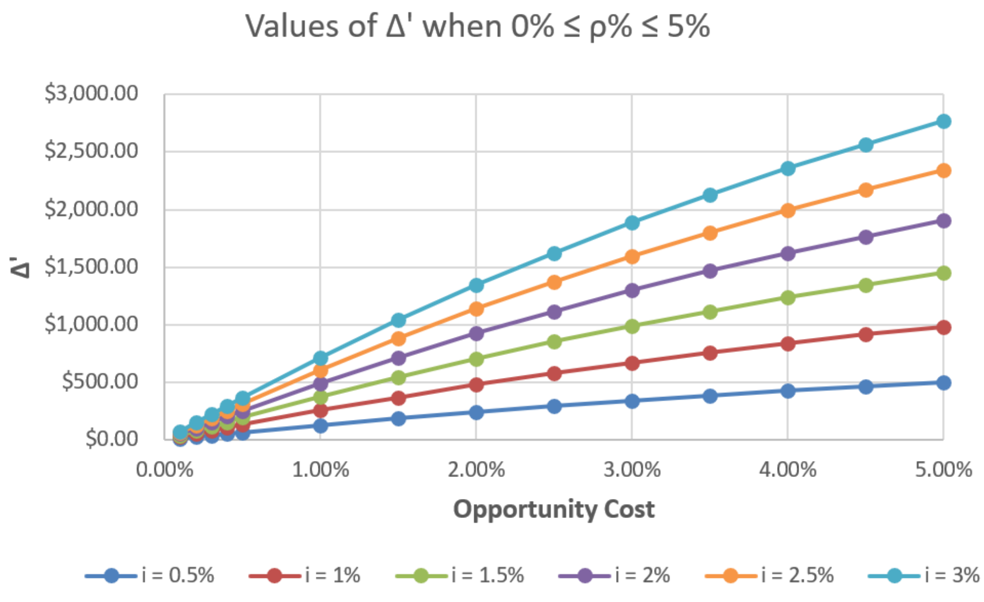

Figure 3. Values of Δ', F =10,000 units of capital and n =12

Like what was seen in the case where the focal date is time *n*, the value of Δ' increases both regarding *i* and ρ.

8. General Analysis

As illustrated in Figure 4, which concerns the case where n = 180 periods, we also have just one change of sign in the sequence d'_k.

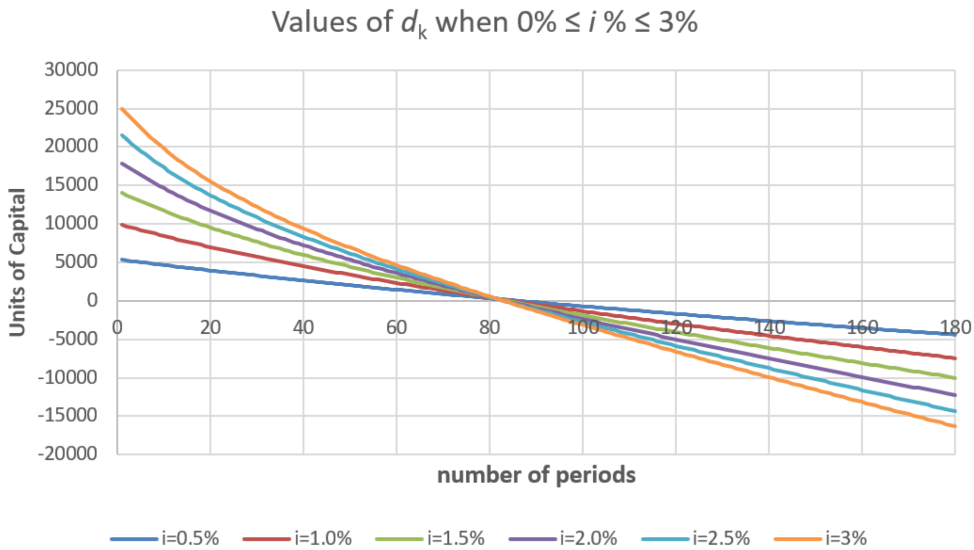

Figure 4. Values of d'_k, F =1,200,000 units of capital and n =180

Consequently, we have a clear indication that we always have $\Delta' > 0$. This means that, also in this case, the financial institution should prefer to use multiple contracts instead of one.

To give a numerical evidence of the values of the fiscal gain, Tables 7, 8, 9, and 10 provide the percentual values of the increase of the fiscal gain $\delta' = \left[V_1'(\rho)\ V_2'(\rho) - 1\right] \times 100$, where ρ_a expresses the annual value of the opportunity cost, and where n_a expresses the length of the contract in years.

Table 7. Fiscal gain δ' – beginning of the contract as the focal date – i = 0.5% p.m.

	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	7.9281	16.0367	24.2981	32.6856	41.1739	49.7393
10	15.9538	33.2326	51.6554	71.0188	91.1122	111.7302
15	23.9891	51.1469	80.8754	112.4668	145.2145	178.4932
20	32.0439	69.5111	110.9019	154.5412	198.9710	243.1316
25	40.1089	88.0151	140.6700	195.0802	249.1306	301.6619
30	48.1636	106.3493	169.3005	232.6985	294.2170	353.0168

Table 8. Fiscal gain δ' – beginning of the contract as the focal date – i = 1.0% p.m.

	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	7.6262	15.3942	23.2771	31.2494	39.2873	47.3687
10	14.9886	31.0182	47.8982	65.4285	83.4132	101.6706
15	22.1519	46.6546	72.8767	100.1569	127.8964	155.6110
20	29.1850	62.1455	97.3722	133.4309	169.2364	204.1086
25	36.1078	77.2767	120.6272	163.8433	205.5881	245.3177
30	42.9188	91.8408	142.1200	190.7724	236.6445	279.6172

Table 9. Fiscal gain *δ'* – beginning of the contract as the focal date – *i* = 1.5% p.m.

	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	7.3833	14.8785	22.4595	30.1019	37.7832	45.4829
10	14.2973	29.4428	45.2434	61.5053	78.0460	94.7024
15	20.9334	43.7102	67.6969	92.2779	116.9324	141.2672
20	27.3922	57.6059	89.1749	120.8409	151.7456	181.4197
25	33.7063	70.9745	109.1140	146.2283	181.3993	214.3857
30	39.8807	83.6635	127.1743	168.1708	206.0717	241.0769

Table 10. Fiscal gain *δ'* – beginning of the contract as the focal date – *i* = 2 % p.m.

	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	7.1817	14.4513	21.7835	29.1548	36.5441	43.9321
10	13.7684	28.2432	43.2318	58.5473	74.0187	89.4981
15	20.0453	41.5815	63.9828	86.6735	109.1916	131.2090
20	26.1290	54.4427	83.5267	112.2550	139.9251	166.2039
25	32.0564	66.7057	101.4200	134.5940	165.5761	194.3068
30	37.8349	78.2486	117.4267	153.6106	186.5633	216.6636

Differently from the case of focal date at the end of the financing period, the fiscal gains are unalike for distinct interest rates. They decrease as interest rates increase (Figure 5) and increase as financing periods (Figure 6) and opportunity cost increase (Figure 7).

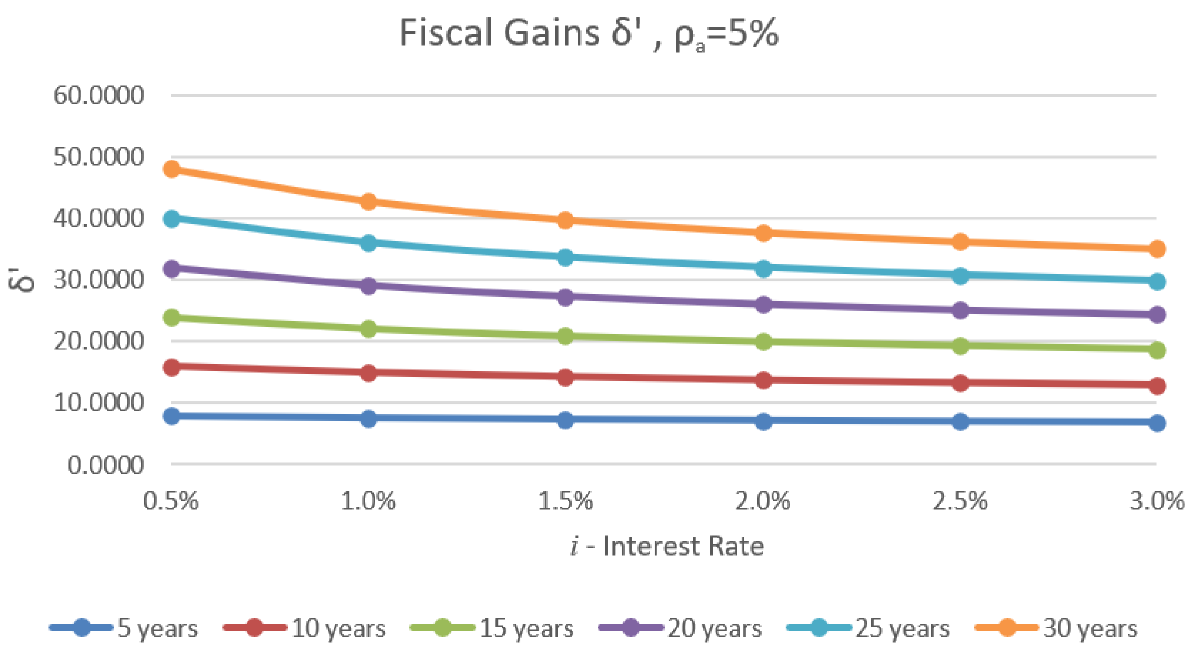

Figure 5. Fiscal gains δ' – ρ_a=5%

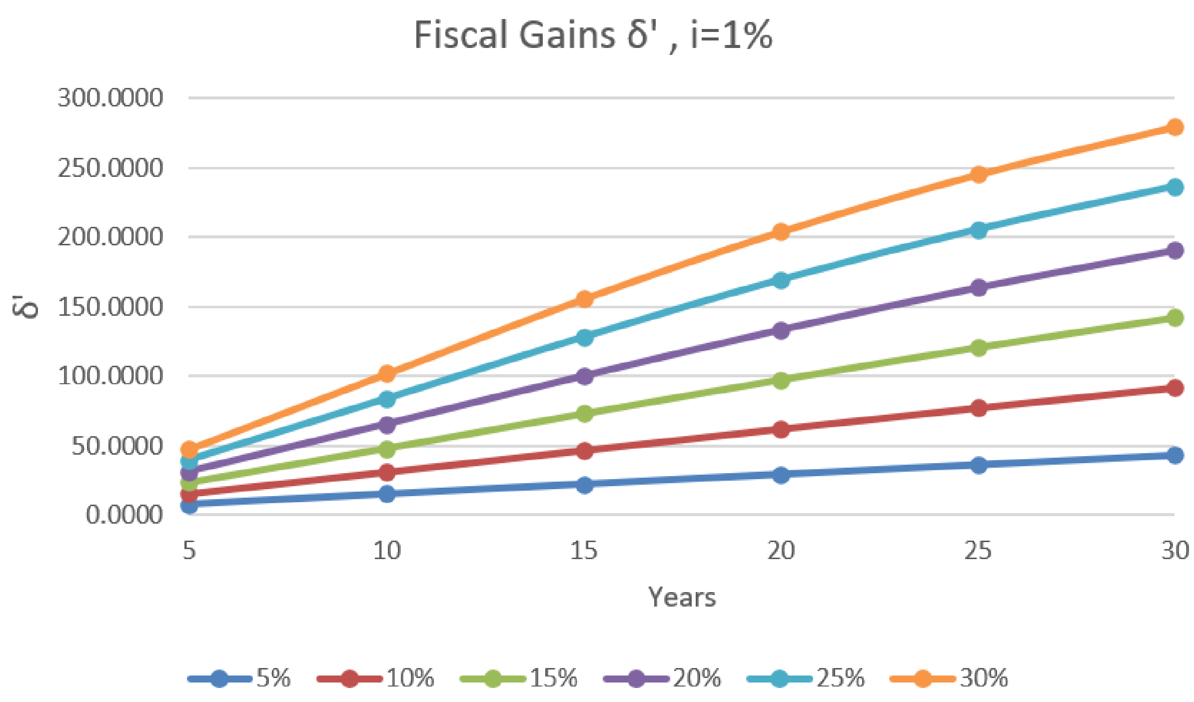

Figure 6. Fiscal gains δ' – i = 1% p.p.

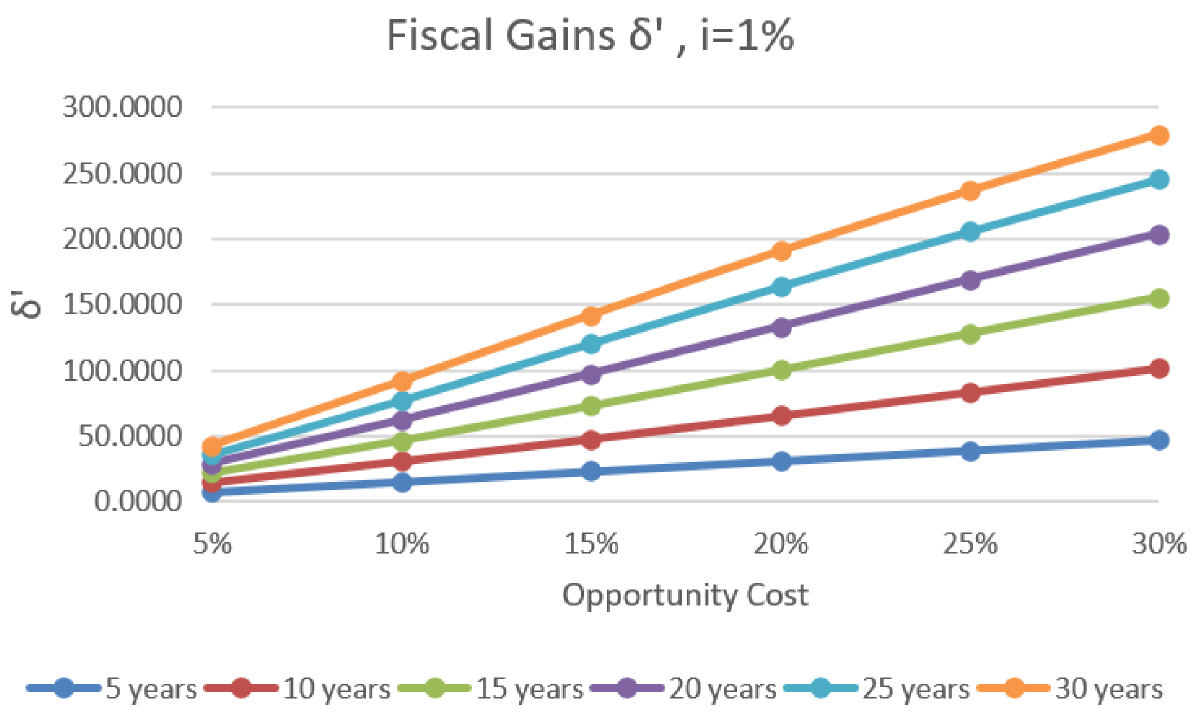

Figure 7. Fiscal gains δ' – i = 1% p.p.

9. Conclusions

Given that, repeatedly, the Brazilian judicial system has determined that home-financing contracts written in terms of compound interest should be substituted by contracts making use of simple interest, we have analyzed the possibility that the financing institution granting the loan decides to substitute a single contract by n subcontracts - one for each one of the payments of the single contract.

Focusing attention on the case of constant payments, which is the most employed, and which in Brazil is known as "Tabela Price," we have concluded that the financing institution granting the loan should always prefer the multiple contracts option since this can result in significant tax gains.

It was shown that the tax gains are higher if the focal date is the end period of the contract, which is the usual case, even though it violates a Brazilian law of 1964.

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Appendices A

For a Single Contract – Constant Payments

$$f = \frac{2}{2 + i \times (n-1)} \tag{A1}$$

$$P^n = \left[\frac{C \times (1-f)}{n}\right] + \left[\left(\frac{C \times f \times i}{2}\right) \times \left(\frac{1+n}{n}\right)\right] = \left[\frac{C}{n} \times (1-f)\right] + \left[\frac{C}{n} \times \frac{(f \times i) \times (1+n)}{2}\right]$$

$$P^n = \frac{C}{n} \times \left[(1-f) + \frac{(f \times i) \times (1+n)}{2}\right]$$

$$= \frac{C}{n} \times \left\{\left[1 - \left(\frac{2}{2 + i \times (n-1)}\right)\right] + \left(\frac{2}{2 + i \times (n-1)}\right) \times \left(\frac{i \times (1+n)}{2}\right)\right\}$$

$$P^n = \frac{C}{n} \times \left\{\left[1 - \left(\frac{2}{2 + i \times (n-1)}\right)\right] + \left(\frac{i \times (1+n)}{2 + i \times (n-1)}\right)\right\} = \frac{C}{n} \times \left\{\left[\left(\frac{i \times (n-1) + i \times (1+n)}{2 + i \times (n-1)}\right)\right]\right\}$$

$$P^n = \frac{C}{n} \times \left(\frac{2 \times i \times n}{2 + i \times (n-1)}\right) \tag{A2}$$

$$A^C = P^C = \left[\frac{C}{n} \times \frac{1}{1 + \frac{i \times (n-1)}{2}}\right] = \frac{C}{n} \times \left[\frac{2}{2 + i \times (n-1)}\right] \tag{A3}$$

$$P = P^C + P^N = \frac{C}{n} \times \left[\frac{2}{2 + i \times (n-1)}\right] + \frac{C}{n} \times \left[\frac{2 \times i \times n}{2 + i \times (n-1)}\right] = \frac{C}{n} \times \left[\frac{2 + (2 \times i \times n)}{2 + i \times (n-1)}\right] \tag{A4}$$

$$J_1 = i \times S_0^C = i \times C \times f = i \times C \times \left[\frac{2}{2 + i \times (n-1)}\right]$$

$$J_2 = i \times S_1^C = i \times (S_0^C - A^C) = i \times \left\{C \times \left[\frac{2}{2 + i \times (n-1)}\right] - \frac{C}{n} \times \left[\frac{2}{2 + i \times (n-1)}\right]\right\}$$

$$J_2 = i \times C \times \left[\frac{2n-2}{n \times [2 + i \times (n-1)]}\right]$$

$$J_3 = i \times S_2^C = i \times (S_0^C - 2 \times A^C) = i \times \left\{C \times \left[\frac{2}{2 + i \times (n-1)}\right] - 2 \times \frac{C}{n} \times \left[\frac{2}{2 + i \times (n-1)}\right]\right\}$$

$$J_3 = i \times C \times \left[\frac{2n-4}{n \times [2 + i \times (n-1)]}\right]$$

:

$$J_k = i \times S_{k-1}^C = i \times (S_0^C - (k-1) \times A^C) = i \times \left\{C \times \left[\frac{2}{2 + i \times (n-1)}\right] - (k-1) \times \frac{C}{n} \times \left[\frac{2}{2 + i \times (n-1)}\right]\right\}$$

$$J_k = i \times C \times \left[\frac{2n - 2 \times (k-1)}{n \times \left[2 + i \times (n-1)\right]}\right] \tag{A5}$$

Sum of Interest – Single Contract

$$\sum_{k=1}^{n} J_k = \sum_{k=1}^{n} \left\{i \times C \times \left[\frac{2n - 2 \times (k-1)}{n \times [2 + i \times (n-1)]}\right]\right\} = \sum_{k=1}^{n} \left\{2 \times i \times C \times \left[\frac{n-k+1}{n \times [2 + i \times (n-1)]}\right]\right\}$$

$$\sum_{k=1}^{n} J_k = \frac{2 \times i \times C}{n \times [2 + i \times (n-1)]} \times \sum_{k=1}^{n} (n-k+1)$$

$$\sum_{k=1}^{n} J_k = \frac{2 \times i \times C}{n \times [2 + i \times (n-1)]} \times [n + (n-1) + (n-2) + \ldots + 2 + 1]$$

$$\sum_{k=1}^{n} J_k = \frac{2 \times i \times C}{n \times [2 + i \times (n-1)]} \times \left[n + \left(\frac{n \times (n-1)}{2}\right)\right] = \frac{2 \times i \times C}{[2 + i \times (n-1)]} \times \left[1 + \left(\frac{(n-1)}{2}\right)\right]$$

$$\sum_{k=1}^{n} J_k = \frac{2 \times i \times C}{[2 + i \times (n-1)]} \times \left[\left(\frac{2 + (n-1)}{2}\right)\right]$$

$$\sum_{k=1}^{n} J_k = \frac{i \times C \times (n+1)}{\left[2 + i \times (n-1)\right]} \tag{A6}$$

Present Value of the k-th interest payment

$$V_1(J_k^{\rho}) = i \times C \times \left[\frac{2n - 2 \times (k-1)}{n \times [2 + i \times (n-1)]}\right] \times \frac{1}{(1+\rho)^k} \tag{A7}$$

Present Value of the interest payment sequence

$$V_1(\rho) = \sum_{k=1}^{n} \left\{ i \times C \times \left[\frac{2n - 2 \times (k-1)}{n \times [2 + i \times (n-1)]}\right] \times \frac{1}{(1+\rho)^k} \right\}$$

$$V_1(\rho) = \frac{2 \times i \times C}{n \times [2 + i \times (n-1)]} \times \sum_{k=1}^{n} \left\{\frac{n - k + 1}{(1+\rho)^k}\right\}$$

$$V_1(\rho) = \frac{2 \times i \times C}{n \times [2 + i \times (n-1)]} \times \left[\frac{n}{(1+\rho)} + \frac{n-1}{(1+\rho)^2} + \ldots + \frac{2}{(1+\rho)^{n-1}} + \frac{1}{(1+\rho)^n}\right] \tag{A8}$$

n subcontracts scheme

$$\hat{P}_k = \hat{P} = P = \frac{C}{n} \times \left[\frac{2 + (2 \times i \times n)}{2 + i \times (n-1)}\right] \tag{A9}$$

k-th principal, $\hat{F}_k$, and *k-th amortization,* $\hat{A}_k$.

$$\hat{F}_k = \hat{A}_k = \frac{\hat{P}_k \times [1 + i \times (n-k)]}{(1 + i \times n)} = \frac{C}{n} \times \left[\frac{2 + (2 \times i \times n)}{2 + i \times (n-1)}\right] \times \frac{[1 + i \times (n-k)]}{(1 + i \times n)} \tag{A10}$$

$$\hat{J}_k = \hat{P}_k - \hat{A}_k = P - \hat{A}_k = P - \frac{P \times [1 + i \times (n-k)]}{(1 + i \times n)} = P \times \left\{1 - \frac{[1 + i \times (n-k)]}{(1 + i \times n)}\right\}$$

$$\hat{J}_k = P \times \left\{\frac{(1 + i \times n) - [1 + i \times (n-k)]}{(1 + i \times n)}\right\}$$

$$\hat{J}_k = P \times \left[\frac{i \times k}{(1 + i \times n)}\right] \tag{A11}$$

Sum of Interest – n Subcontracts

$$\sum_{k=1}^{n} \hat{J}_k = \sum_{k=1}^{n} \left\{ P \times \left[\frac{i \times k}{(1 + i \times n)}\right] \right\} = \frac{P \times i}{(1 + i \times n)} \times \sum_{k=1}^{n} k$$

$$\sum_{k=1}^{n} \hat{J}_k = \frac{P \times i}{(1 + i \times n)} \times \left[1 + 2 + \ldots + (n-1) + n\right] = \frac{P \times i}{(1 + i \times n)} \times \left[n + \frac{n \times (n-1)}{2}\right]$$

$$\sum_{k=1}^{n} \hat{J}_k = \frac{P \times i \times n}{(1 + i \times n)} \times \left[\frac{n+1}{2}\right]$$

$$\sum_{k=1}^{n}\hat{J}_k = \frac{P\times i\times n}{(1+i\times n)}\times\left[\frac{n+1}{2}\right] = \frac{C}{n}\times\left[\frac{2+(2\times i\times n)}{2+i\times(n-1)}\right]\times\frac{i\times n}{(1+i\times n)}\times\left[\frac{n+1}{2}\right]$$

$$\sum_{k=1}^{n}\hat{J}_k = C\times\left[\frac{1+(i\times n)}{2+i\times(n-1)}\right]\times\frac{i}{(1+i\times n)}\times(n+1)$$

$$\sum_{k=1}^{n}\hat{J}_k = C\times\left[\frac{1+(i\times n)}{2+i\times(n-1)}\right]\times\frac{i}{(1+i\times n)}\times(n+1)$$

$$\sum_{k=1}^{n}\hat{J}_k = \left[\frac{i\times C\times(n+1)}{2+i\times(n-1)}\right] \tag{A12}$$

Comparing equations (A6) and (A12).

$$\sum_{k=1}^{n}\hat{J}_k = \left[\frac{i\times C\times(n+1)}{2+i\times(n-1)}\right] = \sum_{k=1}^{n}J_k \tag{A13}$$

This proves that there is no accounting gain in any of the schemes for the client or the financial institution, from this point of view, independent of the interest rate, *i*, number of periods, *n*, or financing capital, *C*.
Present Value of the *k*-th contract interest.

$$V_2(\hat{J}_k^{\rho}) = \frac{C}{n}\times\left[\frac{2+(2\times i\times n)}{2+i\times(n-1)}\right]\times\left[\frac{i\times k}{(1+i\times n)}\right]\times\frac{1}{(1+\rho)^k} \tag{A14}$$

$$V_2(\rho) = \sum_{k=1}^{n}\left\{\frac{C}{n}\times\left[\frac{2+(2\times i\times n)}{2+i\times(n-1)}\right]\times\left[\frac{i\times k}{(1+i\times n)}\right]\times\frac{1}{(1+\rho)^k}\right\}$$

$$V_2(\rho) = \frac{C}{n}\times\left[\frac{2\times(1+i\times n)}{2+i\times(n-1)}\times\frac{1}{(1+i\times n)}\right]\times\sum_{k=1}^{n}\left[\frac{i\times k}{(1+\rho)^k}\right]$$

$$V_2(\rho) = \frac{C}{n}\times\left[\frac{2\times i}{2+i\times(n-1)}\right]\times\sum_{k=1}^{n}\left[\frac{k}{(1+\rho)^k}\right]$$

$$V_2(\rho) = \frac{2\times i\times C}{n\times\left[2+i\times(n-1)\right]}\times\left[\frac{1}{(1+\rho)}+\frac{1}{(1+\rho)^2}+\frac{1}{(1+\rho)^3}+\ldots+\frac{1}{(1+\rho)^n}\right] \tag{A15}$$

Calculating the fiscal gain $\delta = V_1(\rho)/V_2(\rho)-1$

$$\delta = \left\{\frac{\frac{2\times i\times C}{n\times[2+i\times(n-1)]}\times\left[\frac{n}{(1+\rho)}+\frac{n-1}{(1+\rho)^2}+\ldots+\frac{2}{(1+\rho)^{n-1}}+\frac{1}{(1+\rho)^n}\right]}{\frac{2\times i\times C}{n\times[2+i\times(n-1)]}\times\left[\frac{1}{(1+\rho)}+\frac{1}{(1+\rho)^2}+\frac{1}{(1+\rho)^3}+\ldots+\frac{1}{(1+\rho)^n}\right]}\right\}-1$$

$$\delta = \left\{\frac{\left[\frac{n}{(1+\rho)}+\frac{n-1}{(1+\rho)^2}+\ldots+\frac{2}{(1+\rho)^{n-1}}+\frac{1}{(1+\rho)^n}\right]}{\left[\frac{1}{(1+\rho)}+\frac{1}{(1+\rho)^2}+\frac{1}{(1+\rho)^3}+\ldots+\frac{1}{(1+\rho)^n}\right]}\right\}-1$$

As demonstrated, in the case of constant installment systems with a focal date at the end of the financing period, the fiscal gain δ is independent of the interest rate i and the amount of the financing, C, varying only with the number of periods, n, and the opportunity cost, ρ.

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