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An Alternative Multiple Contracts Version of SACRE

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Abstract

1. Introduction

In 1996, Caixa Econômica Federal (CEF), which is the foremost house-financing institution in Brazil with a history of innovation, introduced a new debt amortization scheme named "Sistema de Amortizações Reais Crescentes" – SACRE (system of increasing amortizations in real terms).

As the CEF original version is not financially consistent, two distinct procedures have been suggested to remedy this deficiency.

The first one was developed by Forger (2010) and was based on the well-established principle which states that the present value of the sequence of the corresponding payments has to equal the value of the principal F being financed, taking into account the rate of interest.

The second one named SACRE*, however, proposed by de Faro and Lachtermacher (2022), was devised in such a way as to maintain the original CEF proposal as close as possible.

Given that de Faro and Lachtermacher (2023) has already proposed a multiple contracts version of the SACRE*, we will present a multiple contracts version of the Forger (2010) proposal, namely SACRE-F.

2. The Single Contract Version of the SACRE-F Procedure

Denoting by F the loan amount, and by i the periodic interest rate being charged, suppose that in the case where a single contract is considered, it has been stipulated that the debt has to be reimbursed by n periodic payments with the k-th payment being denoted as $\hat{P}_k$, since p_k will denote the k-th payment in the case of implementation of SACRE*.

Like the case of the original CEF scheme, the number n of the periodic payments is divided into ℓ subperiods, each one with m constant payments.

Fundamentally, it is supposed that:

$$F = \sum_{k=1}^{n} \hat{P}_k \times (1+i)^{-k} \tag{1}$$

or, equivalently, in compound interest capitalization

$$F \times (1+i)^n = \sum_{k=1}^{n} \hat{P}_k \times (1+i)^{n-k} \tag{1'}$$

Furthermore, as also proposed by Forger (2010), it is convenient to decompose the index k, $1 \le k \le n$, that identifies the k-th payment $\hat{P}_k$, according to ℓ and m, as follows:

$$k = (p-1) \times m + q \text{, with } 1 \le p \le \ell \text{ and } 1 \le q \le m \tag{2}$$

With this proviso, we can simplify the notation in such a way that:

$$\hat{P}_k = \hat{P}_{(p-1)\times m+q} = \hat{P}'_p \text{, for any } p,\ 1 \le p \le \ell \text{, and any } q,\ 1 \le q \le m \tag{3}$$

or, more explicitly, it is supposed that the payments are constant in each subperiod:

$$\hat{P}_{(p-1)\times m+1} = \hat{P}_{(p-1)\times m+2} = \cdots = \hat{P}_{(p-1)\times m+m} = \hat{P}_p' \tag{3'}$$

With this notation, it follows that expression (1') can be rewritten as:

$$F \times (1+i)^n = \sum_{p=1}^{\ell}\sum_{q=1}^{m} \hat{P}_{(p-1)\times m+q} \times (1+i)^{\ell\times m-(p-1)\times m-q}$$

or, after some simple manipulations

$$F \times (1+i)^n = \left[\frac{(1+i)^m-1}{i}\right] \times \sum_{p=1}^{\ell} \hat{P}_p' \times [(1+i)^m]^{\ell-p} \tag{1''}$$

Additionally, Forger (2010) also proposed that, from any subperiod to the next, there is a linear increase on the values of the constant payments. It was then derived that:

$$\hat{P}_k = \hat{P}_{(p-1)\times m+q} = \hat{P}_p' = \left(\frac{F\times i}{\ell}\right) \times \left[(\ell-p+1) + \frac{1}{(1+i)^m-1}\right] \tag{4}$$

In summary, the sequence of payments when implementing the Forger version of SACRE is:

$$\hat{P}_k = \begin{cases} P_1' = \left(\frac{F\times i}{\ell}\right) \times \left[(\ell) + \frac{1}{(1+i)^m-1}\right],\ k = 1,2,\dots,m \\ P_2' = \left(\frac{F\times i}{\ell}\right) \times \left[(\ell-1) + \frac{1}{(1+i)^m-1}\right], k = m+1, m+2, \dots, 2m \\ \vdots \\ P_\ell' = \left(\frac{F\times i}{\ell}\right) \times \left[1 + \frac{1}{(1+i)^m-1}\right],\ k = (\ell-1)\times m, (\ell-2)\times m, \dots, n \end{cases} \tag{5}$$

Before proceeding, it is interesting to point out, as shown in Forger (2010), that expression (4) encompasses two very particular and important cases. For instance, when $\ell = p = 1$ and $m = n$, we have the case of constant payments. While, if $\ell = n$, $p = k$ and $m = q = 1$, we have the case of constant amortization.

With regard to the evolution of the outstanding debt at time k, also following Forger (2010), we have $k = 1, 2, \dots, n$ and $S_0 = F$:

$$\hat{S}_k = \hat{S}_{(p-1)\times m+q} = \frac{F}{\ell} \times \left[(\ell-p+1) - \frac{(1+i)^q-1}{(1+i)^m-1}\right] \tag{6}$$

with the parcels of amortization and of interest, which comprises the payment $\hat{P}_k$ being respectively given as:

$$\hat{A}_k = \hat{A}_{(p-1)\times m+q} = \frac{F\times i}{\ell} \times \left[\frac{(1+i)^q-1}{(1+i)^m-1}\right] \tag{7}$$

and

$$\hat{J}_k = \hat{J}_{(p-1)\times m+q} = \frac{F\times i}{\ell} \times \left[(\ell-p+1) - \frac{(1+i)^{q-1}-1}{(1+i)^m-1}\right] \tag{8}$$

3. A numerical Example

Fixing the number of payments $n = 12$, with $\ell = 4$ and $m = 3$, we present in Table 1 the corresponding sequences of $\hat{P}_k$, $\hat{S}_k$ and of $\hat{J}_k$. Additionally, to make a numerical comparison with the case of SACRE*, Table 1 also presents the corresponding values of the sequence of payments p_k, the sequence of the values of the outstanding debt S_k, and also the sequence of the parcels of interest J_k. In both cases, $F = 12{,}000$ units of capital and $i = 1\%$ per period.

Table 1. Numerical Comparison between SACRE-F and SACRE*

k	$\hat{P}_k$	$\hat{S}_k$	$\hat{J}_k$	p_k	S_k	J_k
0	----	12,000.00	----	----	12,000.00	----
1	1,110.07	11,009.93	120.00	1,120.00	11,000.00	120.00
2	1,110.07	10,009.97	110.10	1,120.00	9,990.00	110.00
3	1,110,07	9,000.00	100.10	1,120.00	8,969.90	99.90
4	1,080.07	8,009.93	90.00	1,086.35	7,973.24	89.70
5	1,080.07	7,009.97	80.10	1,086.35	6,966.62	79.73
6	1,080.07	6,000.00	70.10	1,086.35	5,949.93	69.67
7	1,050.07	5,009.93	60.00	1,051.16	4,958.28	59.50
8	1,050.07	4,009.97	50.10	1,051.16	3,956.71	49.58
9	1,050.07	3,000,00	40.10	1,051.16	2,945.12	39.57
10	1,020.07	2,009.93	30.00	1,011.16	1,963.41	29.45
11	1,020.07	1,009.97	20.10	1,001.34	981.71	19.63
12	1,020.07	0.00	10.10	991.52	0.00	9.82
Total	12,780.80	----	780.80	12,776.55	----	776.55

As it should be expected, both the SACRE-F and the SACRE* are confirmed to be financially consistent. That is, both procedures lead to full amortization of the debt.

On the other hand, although $p_1 > \hat{P}_1$, we have that $\sum_{k-1}^{n} \hat{P}_k > \sum_{k-1}^{n} p_k$. Therefore, the debtor must pay more interest in the case of SACRE-F.

Moreover, this appears to be a general result. Since, as illustrated in Figure 1, whereas *F* equals to 1,200,000 units of capital, to magnify the numerical differences, and *n* = 240 months (20 years), we have that:

$$\Delta = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k} - \sum_{k=1}^{n} J_k \times (1+\rho)^{-k} = V_1(\rho) - V_2(\rho) > 0 \quad (9)$$

where $\boldsymbol{\rho}$, supposed to be relative to the same period as the interest rate $\boldsymbol{i}$, denotes the financial institution cost of capital.

It should be noted that, in Figure 1, the interest rate $\boldsymbol{i}$ is monthly, while the opportunity cost, identified as ρ_a, is expressed in annual terms.

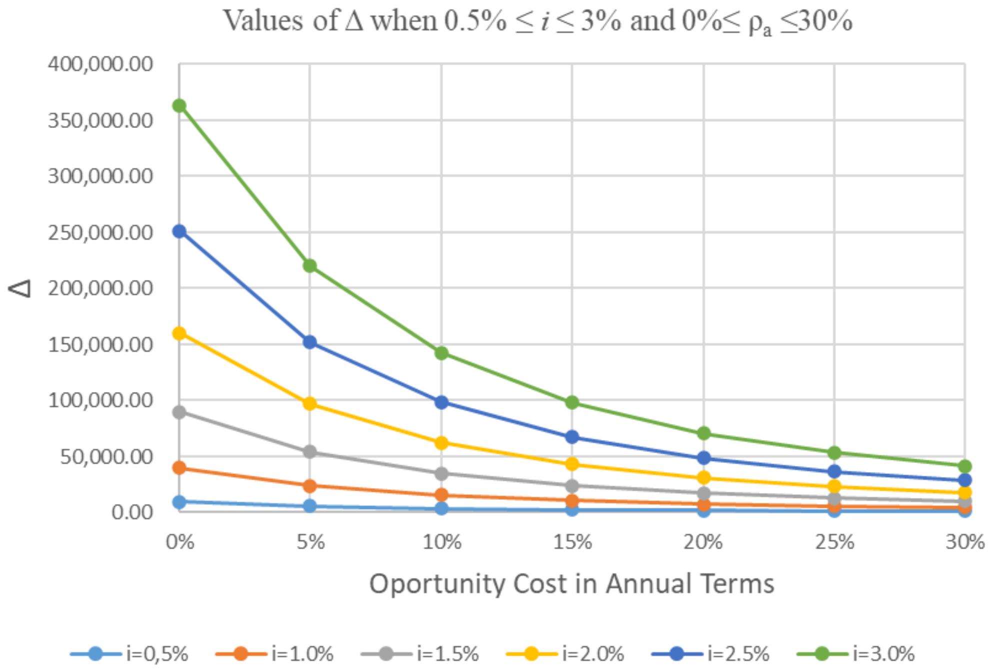

Figure 1. Numerical Differences Δ, F =1,200,000, *n* = 240 months

In Tables 2 and 3, which refer to the cases where i = 0.5% monthly and i =1% monthly, respectively, we have the percentual increase of the fiscal gain $\delta = \left[V_1(\rho_a)\ V_2(\rho_a) - 1\right] \times 100$ where ρ_a expresses the annual value of the opportunity cost, and where n_a expresses the length of the contract in years.

Table 2. Fiscal Gains δ – Single Contract – i = 0.5% p.m.

	ρ_a					
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	1.4950	1.4149	1.3423	1.2765	1.2165	1.1619
10	1.3467	1.2087	1.0914	0.9915	0.9061	0.8330
15	1.2468	1.0636	0.9179	0.8017	0.7086	0.6334
20	1.1234	0.9233	0.7715	0.6563	0.5680	0.4995
25	1.0170	0.8086	0.6582	0.5492	0.4688	0.4083
30	0.9257	0.7148	0.5698	0.4689	0.3968	0.3437

Table 3. Fiscal Gains δ – Single Contract – i = 1.0% p.m.

	ρ_a					
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	3.0462	2.8815	2.7325	2.5973	2.4745	2.3625
10	2.7397	2.4575	2.2179	2.0142	1.8403	1.6914
15	2.5350	2.1613	1.8644	1.6281	1.4388	1.2858
20	2.2859	1.8773	1.5678	1.3332	1.1536	1.0144
25	2.0699	1.6444	1.3378	1.1158	0.9523	0.8293
30	1.8844	1.4538	1.1583	0.9528	0.8061	0.6981

The results presented, which are even greater if the interest rate i is increased, confirm that the financing institution, when writing a single contract, should always choose the SACRE* version.

4. The Multiple Contracts Alternative

Instead of a single contract, the financial institution has the option of requiring the borrower to write n subcontracts - one for each of the n payments that would be associated with the case of a single contract, with the principal of the k-th subcontract being the present value, at the same interest rate i, of the k-th payment of the single contract.

That is, the principal of the k-th subcontract, denoted by $\hat{F}_k$, is:

$$\hat{F}_k = \hat{P}_k \times (1+i)^{-k}, \quad k = 1,2,\dots,n \tag{10}$$

In this case, the parcel of amortization associated with the k-th payment, denoted by $\hat{A}'_k$, will be:

$$\hat{A}'_k = \hat{F}_k = \hat{P}_k \times (1+i)^{-k}, \quad k = 1,2,\dots,n \tag{11}$$

Namely, the parcel of amortization associated with the k-th subcontract is exactly equal to the value of the corresponding principal.

On the other hand, from an accounting point of view, it follows that the parcel of interest associated with the k-th subcontract, which will be denoted by $\hat{J}'_k$, wherein:

$$\hat{J}'_k = \hat{P}_k \times \{1 - (1+i)^{-k}\} = \hat{P}_k - \hat{F}_k, \quad k = 1,2,\dots,n \tag{12}$$

From the strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the subcontracts, the total interest payment is the same in both cases. However, in terms of present values, and depending on the financial institution's opportunity cost, it is possible that the financial institution will be better off if adopts the option of multiple contracts.

4.1 A Simple Numerical Example

Considering the same simple numerical example in section 3, but now fixing F=120,000 units of capital, Table 4 presents the values of the sequence of payments $\hat{P}_k$, the sequence of the parcels of interest, $\hat{J}_k$ in the case of a single contract, and the sequence of the parcels of interest $\hat{J}'_k$, in the case of multiple contracts, as well as the sequence of differences, $d_k = \hat{J}_k - \hat{J}'_k$.

Table 4. Sequences of the Parcels of Interest and its Differences

k	$\hat{P}_k$	$\hat{J}_k$	$\hat{J}'_k$	d_k
1	11,100.66	1,200.00	109.91	1,090.09
2	11,100.66	1,100.99	218.73	882.27
3	11,100.66	1,001.00	326.47	674.53
4	10,800.66	900.00	421.44	478.56
5	10,800.66	800.99	524.20	276.79
6	10,800.66	701.00	625.95	75.05
7	10,500.66	600.00	706.51	**-106.51**
8	10,500.66	500.99	803.48	**-302.48**
9	10,500.66	401.00	899.49	**-498.49**
10	10,200.66	300.00	966.14	**-666.14**
11	10,200.66	200.99	1,057.57	**-856.57**
12	10,200.66	101.00	1,148.09	**-1,047.10**
Total	127,807.96	7,807.96	7,807.96	0.00

The sequence of differences d_k has only one change of sign, thus characterizing what is defined a conventional financing project, cf. de Faro (1974), which internal rate of return is known to be unique, and, in this case, is equal to zero.

Therefore, we are assured that:

$$\hat{\Delta} = V_1(\rho) - V_3(\rho) = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k} - \sum_{k=1}^{n} \hat{J}'_k \times (1+\rho)^{-k} > 0 \tag{13}$$

for all $\rho > 0$.

That is, at least in the case of our simple numerical example, the financial institution granting the loan will be better off if it adopts the multiple contracts option.

5. A General Analysis

In the previous section, focusing attention on the case of a contract with only 12 payments, it was verified that the sequence d_k of differences of the interest payments has just one change of sign, thereby assuring us of the uniqueness of the corresponding internal rate of return, which was known to be zero.

However, when the number of payments is increased, it is possible to have instances where more than one change of sign can occur in the sequence d_k.

This possibility is illustrated in Figure 2, which refers to the case where the contract has a term of 15 years, with $\ell = 15$ and monthly payments, with the monthly interest rate i going from 0.5% up to 3%, and with the value of the loan amount still fixed at 120,000 units of capital as well.

While we have only one change of sign if the interest rate is 0.5%, 1.5%, 2.5% or 3%, we have 3 changes of sign if is 1% or 2%.

However, considering a classical result first stated by Norstrom (1972), which is based on the sequence of the accumulated values of the sequence d_k, we can still guarantee the uniqueness of the corresponding internal rate of return, which value we already know to be zero.

Therefore, in these instances we also have $\hat{\Delta} > 0$ as illustrated in Figure 3, where 0% < ρ ≤ 10% monthly, and with the monthly interest rate i going to 0.5% to 3%.

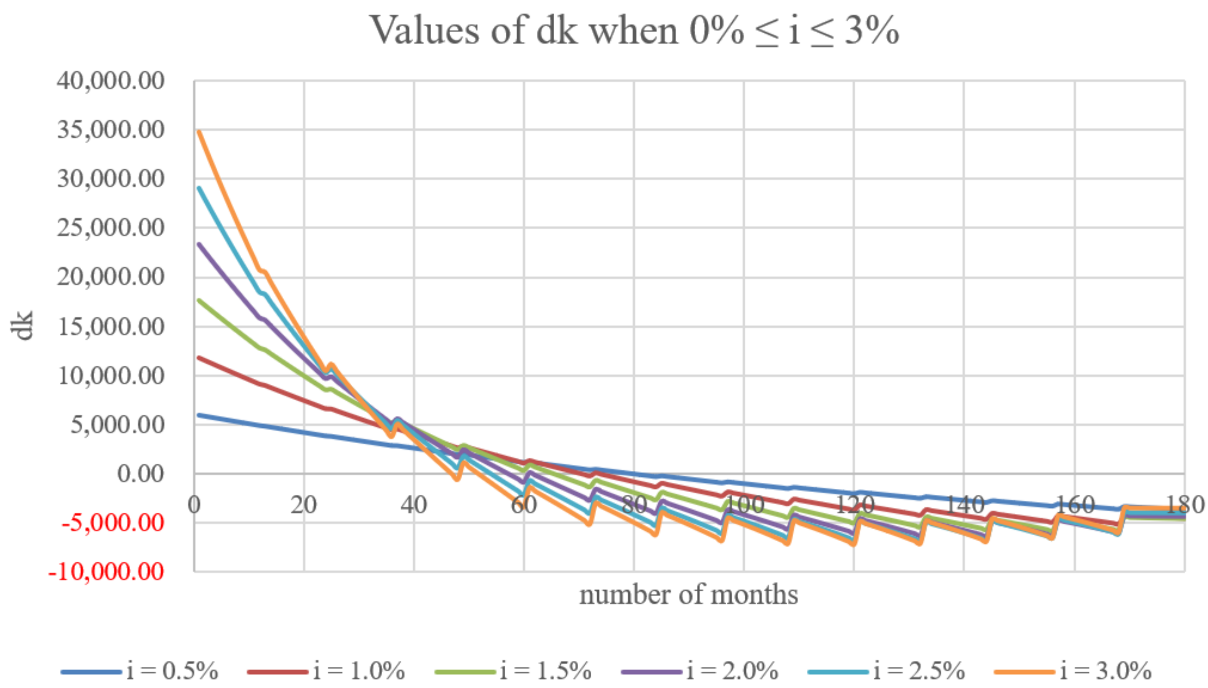

Figure 2. Numerical Differences $\boldsymbol{d_k}$, $\boldsymbol{F}$ =120,000, $\boldsymbol{n}$ = 180 months

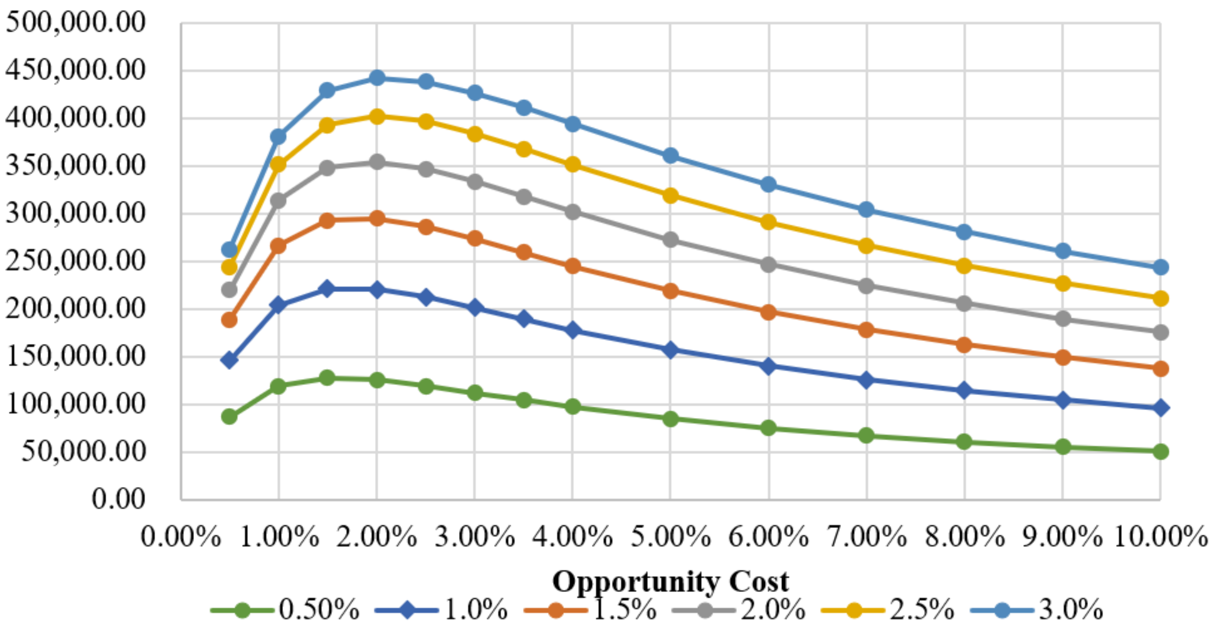

Figure 3. Numerical Differences, when 0% < $\boldsymbol{\rho}$ ≤ 10%

As the results which are shown here appear to be general, Tables 5 to 8 present the percentual increase of the fiscal gain $\delta' = \left[V_1(\rho_a)/V_3(\rho_a) - 1\right] \times 100$, for some values of the corresponding annual opportunity cost $\boldsymbol{\rho_a}$, with each contract with a term of $\boldsymbol{n_a}$ years, subdivided in $\boldsymbol{\ell} = \boldsymbol{n_a}$ subperiods, and each one with $\boldsymbol{m}$ = 12 monthly payments.

Table 5. Fiscal Gain δ' - SACRE-F Single Contract x SACRE-F Multiple Contracts – *i*=0.5%p.m.

			ρ_a			
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	7.6901	15.5327	23.5009	31.5692	39.7136	47.9118
10	14.8754	30.8053	47.6112	65.1048	83.0997	101.4217
15	21.3935	45.0474	70.4115	96.9078	124.0058	151.2661
20	27.2449	57.9215	90.8261	124.8107	158.9760	192.7153
25	32.4560	69.2784	108.3876	148.0785	187.2683	225.3972
30	37.0706	79.1213	123.1172	166.9883	209.7072	250.8971

Table 6. Fiscal Gain δ' - SACRE-F Single Contract x SACRE-F Multiple Contracts – i=1.0%p.m.

			ρ_a			
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	7.1358	14.3636	21.6590	28.9997	36.3649	43.7362
10	12.8816	26.3818	40.3413	54.6069	69.0398	83.5197
15	17.4041	35.9326	55.1570	74.6943	94.2369	113.5588
20	20.9680	43.3824	66.4581	89.5839	112.3551	134.5401
25	23.7935	49.1579	74.9480	100.4184	125.1819	149.0817
30	26.0531	53.6454	81.3335	108.3527	134.4075	159.4337

Table 7. Fiscal Gain δ' - SACRE-F Single Contract x SACRE-F Multiple Contracts – *i*=1.5%p.m.

			ρ_a			
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	6.6460	13.3385	20.0559	26.7792	33.4910	40.1764
10	11.3132	22.9823	34.8753	46.8736	58.8761	70.8000
15	14.5795	29.7269	45.1286	60.5308	75.7470	90.6526
20	16.9179	34.4741	52.1571	69.6139	86.6365	103.1217
25	18.6362	37.8759	57.0399	75.7430	93.8135	111.2006
30	19.9314	40.3682	60.5145	80.0092	98.7421	116.7123

Table 8. Fiscal Gain δ' - SACRE-F Single Contract x SACRE-F Multiple Contracts – *i*=2.0%p.m.

			ρ_a			
n_a	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
5	6.2112	12.4343	18.6510	24.8451	31.0024	37.1109
10	10.0556	20.3035	30.6371	40.9656	51.2150	61.3280
15	12.4956	25.2653	38.0789	50.7626	63.1981	75.3123
20	14.1222	28.5095	42.8135	56.8139	70.3930	83.5013
25	15.2585	30.7197	45.9462	60.7135	74.9359	88.5998
30	16.0849	32.2845	48.1062	63.3519	77.9769	91.9978

Therefore, as indicated by the results in Tables 5 to 8, the financial institution should always prefer the multiple contracts version of the SACRE-F scheme.

6. A multiple contracts paradox

In section 3, comparing the SACRE* with the SACRE-F for the case of single contracts, it was shown that the financing institution should always prefer the first. However, and thus leading to a paradox, in the case of multiple contracts this is not always true.

For instance, fixing F = 1,200,000 units of capital, to magnify the numerical differences, if the term of the contract is 15 years, and the financing rate is 0.5% per month, Table 9 presents the value of $\boldsymbol{V_3(\rho)}$ as given in relation (13), and the value of $\boldsymbol{V_4(\rho)}$, defined follows:

$$V_4(\rho) = \sum_{k=1}^{n} p_k \times \{1-(1+i)^{-k}\} \times (1+\rho)^{-k} \tag{14}$$

Additionally, Table 9 also shows the numerical differences of $\hat{\Delta}' = V_3(\rho) - V_4(\rho)$.

Table 9. Values of the numerical differences $\hat{\Delta}'$ when i = 0.5% p.m. and $\boldsymbol{n_a}$ = 15 years

ρ_a	$V_3(\rho_a)$	$V_4(\rho_a)$	$\hat{\Delta}'$
0%	543,356.59	535,452.84	7,903.74
5%	354,822.54	351,070.95	3,751.59
10%	243,987.03	242,267.21	1,719.81
15%	175,621.56	174,919.32	702.24
20%	131,556.91	131,371.24	185.67
25%	101,997.91	102,074.47	**-76.56**
30%	81,442.33	81,649.15	**-206.82**

As can be seen, and thus evidencing a paradox, in the case of multiple contracts, we have situations where the financing institution should prefer the SACRE-F scheme.

For instance, this occurs when either the opportunity cost is 25% in annual terms, or 30%. In both cases, we have $\hat{\Delta}' < 0$, which means that, in the case of adopting multiple contracts, it is not always true that the financing institution is better off if it adopts the SACRE* scheme.

In Table 10, which refers to the case where $\boldsymbol{n_a}$ is 30 years, we also have situations where the financing institution would be better off if the multiple contracts version of the SACRE-F were applied.

Table 10. Values of the numerical differences $\hat{\Delta}'$ when i = 0.5% p.m. and $\boldsymbol{n_a}$ = 15 years

ρ_a	$V_3(\rho_a)$	$V_4(\rho_a)$	$\hat{\Delta}'$
0%	1,083,356.59	1,070,220.33	13,136.26
5%	511,500.01	507,783.07	3,716.94
10%	282,189.54	281,252.60	936.93
15%	175,980.16	175,915.80	64.36
20%	120,292.95	120,503.95	**-210.99**
25%	87,977.33	88,264.06	**-286.73**
30%	67,650.94	67,944.47	**-293.52**

Increasing the interest rate *i* to 1% per month, and fixing $\boldsymbol{n_a}$ = 15 years, it can be seen in Table 11 that the SACRE-F is always dominated.

Table 11. Values of the numerical differences $\hat{\Delta}'$ when *i* = 1.0% p.m. and $\boldsymbol{n_a}$ = 15 years

ρ_a	$V_3(\rho_a)$	$V_4(\rho_a)$	$\hat{\Delta}'$
0%	1,087,422.56	1,055,709.71	31,712.85
5%	734,185.84	717,819.56	16,366.28
10%	520,970.02	512,447.11	8,522.91
15%	385,965.08	381,586.34	4,378.74
20%	296,707.02	294,579.74	2,127.28
25%	235,363.57	234,486.48	877.09
30%	191,725.45	191,552.92	172.53

However, if the term of the contract is $\boldsymbol{n_a}$ = 30 years, we also have an instance that can be considered as paradox. Namely, if $\boldsymbol{\rho_a}$ = 30% per annum, we have $\hat{\Delta}' < 0$ as depicted in Table 12.

Table 12. Values of the numerical differences $\hat{\Delta}'$ when i = 1.0% p.m. and $\boldsymbol{n_a}$ = 30 years

ρ_a	$V_3(\rho_a)$	$V_4(\rho_a)$	$\hat{\Delta}'$
0%	2,167,422.56	2,114,511.08	52,911.48
5%	1,112,707.85	1,094,508.72	18,199.13
10%	658,109.26	651,428.90	6,680.37
15%	433,151.54	430,713.40	2,438.14
20%	308,353.73	307,615.78	737.95
25%	232,521.60	232,507.02	14.58
30%	183,036.52	183,338.40	**-301.88**

On the other hand, whenever *i* is greater than 1% per month, it appears that the paradox vanishes establishing that the SACRE* version of multiple contracts is the better choice.

7. Conclusion

Given that the SACRE, as originally proposed by CEF, is not financially consistent, two variants have been proposed.

As shown here, the de Faro and Lachtermacher (2022) SACRE* variant, as analyzed in de Faro and Lachtermacher (2023), appears to be the dominant one, both in the case of a single contract and in the case of multiple contracts.

Notwithstanding, for unusually high values of the financing institution opportunity cost, a multiple contracts version of the SACRE-F proposal may be the dominant one.

Furthermore, if the SACRE-F is chosen to be implemented, the financing institution should always make use of the multiple contract's version.

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