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The G-Qubit Theory Alternative to Conventional Tensor-Product Explanation of Entanglement

Alexander SOIGUINE[1]

[1] SOiGUINE Quantum Computing, USA

Correspondence: Alexander SOIGUINE, SOiGUINE Quantum Computing, Aliso Viejo, CA 92656, USA. Website: https://soiguine.com; E-mail: alex@soiguine.com

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Abstract

Quantum computing rests upon two theoretical pillars: entanglement and superposition. But some physicists say that this is a very shaky foundation and quantum computing success will have to be based on a different theoretical foundation. The g-qubit theory supports this point of view. Current article is the second one of the two and about the entanglement. It gives different, more physically feasible, not mysterious, explanation of what the entanglement is. The suggested formalism demonstrates that the core of future quantum computing should not be in entanglement which only formally follows in conventional quantum mechanics from representation of the many particle states as tensor products of individual states. The core of quantum computing scheme should be in manipulation and transferring of wave functions on $\mathbb{S}^3$ as operators acting on observables and formulated in terms of geometrical algebra. In this way quantum computer will be a kind of analog computer keeping and processing information by sets of objects possessing infinite number of degrees of freedom, contrary to the two value bits or two-dimensional Hilbert space elements, qubits.

Keywords: geometric algebra, wave functions, observables, measurements

1. Introduction

1.1 Entanglement in Conventional Quantum Mechanics

An old Einstein's suggestion was that quantum mechanics is not the root level of reality, but merely hazy glimpse of something even deeper. But another idea may be that quantum mechanics is not of something deeper but should be replaced by something conceptually different.

One of quantum mechanics milestones, **complementarity principle**, says that a complete knowledge of phenomena on atomic dimensions requires a description of both wave and particle properties. The principle was announced in 1928 by the Danish physicist Niels Bohr. His statement was that depending on the experimental arrangement, the behavior of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like and that it is impossible to observe both the wave and particle aspects simultaneously.

A state in the conventional quantum mechanics, assigned to a particle, was said what state can do in making a measurement on the particle.

In the suggested alternative it is said that theory should speak not about complementarity but about proper dividing of the measurement interaction process **into operator, state, or wave function, which is the three-sphere $\mathbb{S}^3$ element acting on observable, and operand, the measured observable.**

A *vector in quantum mechanics* is the mathematical gadget used to describe the *state* of a quantum system, its status, what it's capable of doing. A state assigned to elementary particles there is given by a unit vector in a vector space, really a Hilbert space C^n, particularly C^2, encoding information about the state. The dimension n is the number of different observable things after making a measurement on the particle.

The simplest quantum mechanical state, qubit, reads:

$$C^2 \ni \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = z_1|0\rangle + z_2|1\rangle$$

It has just two observable "things" after measurement, say "up" for $|0\rangle$ and "down" for $|1\rangle$, with probabilities z_1^2 and z_2^2.

In the case of two particles vector space C^2 is generalized to density matrix defined on tensor product $C^2 \otimes C^2$ and in the case of N particles we get $C^2 \otimes C^2 \otimes \ldots \otimes C^2$, N-fold tensor product.

The appropriateness of tensor products is that the tensor product itself captures all ways that basic things can "interact" with each other.

2. Wave Functions in the G-Qubit Theory

The path to the new theory starts with generalization of complex numbers by explicit introduction of a variable "complex" plane in three dimensions that immediately eliminates the questions like "Why do we need imaginary unit in quantum mechanics?" [1]

State, wave function, will be a unit value element of even subalgebra of three-dimensional geometric algebra. Such elements will execute twisting of observables. Even subalgebra G_3^+ is subalgebra of elements of the form $M_3 = \alpha + I_S\beta$, where α and β are (real) (Note 1) scalars and I_S is some unit bivector arbitrary placed in three-dimensional space.

Wave functions as elements of G_3^+ are naturally mapped onto unit sphere $\mathbb{S}^3$ [2], [3], [4].

If in some bivector basis $\{B_1, B_2, B_3\}$, with, for example, right-hand screw multiplication rules $B_1B_2B_3 = 1$, $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, the twisting plane bivector is

$$I_S = b^1B_1 + b^2B_2 + b^3B_3,$$

then

$$\alpha + I_S\beta = \alpha + \beta b^1B_1 + \beta b^2B_2 + \beta b^3B_3$$
$$\alpha + I_S\beta \Rightarrow \{\alpha, \beta b^1, \beta b^2, \beta b^3\}$$

and

$$(\alpha)^2 + (\beta)^2((b^1)^2 + (b^2)^2 + (b^3)^2) = (\alpha)^2 + (\beta)^2 = 1,$$

since wave function is normalized and bivector I_S is a unit value one.

Wave function can always be conveniently written as exponent, see [2], Sec.2.5,:

$$\alpha + I_S\beta = e^{I_S\varphi},\ \alpha = \cos\varphi,\ \ \beta = \sin\varphi$$

The product of two exponents is again an exponent, because generally $|g_1g_2| = |g_1||g_2|$ and $\left|e^{I_{S_1}\alpha}e^{I_{S_2}\beta}\right| = \left|e^{I_{S_1}\alpha}\right|\left|e^{I_{S_2}\beta}\right| = 1$.

Multiplication of an exponent by another exponent is often called *Clifford translation.* Using the term *translation* follows from the fact that Clifford translation does not change distances between the exponents it acts upon if we identify exponents as points on unit sphere $\mathbb{S}^3$*:*

$$\cos\alpha + I_S\sin\alpha = \cos\alpha + b_1\sin\alpha\, B_1 + b_2\sin\alpha\, B_2 + b_3\sin\alpha\, B_3 \Leftrightarrow \{\cos\alpha, b_1\sin\alpha, b_2\sin\alpha, b_3\sin\alpha\}$$
$$(\cos\alpha)^2 + (b_1\sin\alpha)^2 + (b_2\sin\alpha)^2 + (b_3\sin\alpha)^2 = 1$$

This result follows again from $|g_1g_2| = |g_1||g_2|$:

$$|e^{I_S\alpha}(g_1 - g_2)| = |e^{I_S\alpha}||g_1 - g_2| = |g_1 - g_2|$$

Clifford translation of a wave function $e^{I_{S_2}\varphi_2}$ *by* $e^{I_{S_1}\varphi_1}$ is displacement of the wave function, point on $\mathbb{S}^3$**,** along big circle that is intersection of $\mathbb{S}^3$ *by* S_1 by parameter φ_1.

The core of quantum computing scheme should be in manipulation and transferring of wave functions on $\mathbb{S}^3$ as operators acting on observables and formulated in terms of geometrical algebra. In this way quantum computer will be a kind of analog computer keeping and processing information by sets of objects possessing infinite number of degrees of freedom, contrary to the two-dimensional Hilbert space elements, qubits.

3. The Meaning of Schrodinger Equation

Let us take some, generally not normalized, vector $H(t) = I_3(\chi^1(t)B_1 + \chi^2(t)B_2 + \chi^3(t)B_3)$ **(**Note 2**) and execute infinitesimal Clifford translation of a wave function** $e^{I_{S(t)}\varphi(t)}$ **using bivector** $-I_3H(t)$ **and Clifford parameter** $|H(t_0)|\Delta t$ **at some instant of time** t_0**:**

$$e^{-I_3\frac{H(t_0)}{|H(t_0)|}|H(t_0)|\Delta t}e^{I_{S(t_0)}\varphi(t_0)}$$

With denoting $I_3\frac{H(t_0)}{|H(t_0)|} \equiv I_H(t_0)$ **we get:**

$$e^{I_{S(t_0+\Delta t)}\varphi(t_0+\Delta t)} \approx e^{-I_H(t_0)|H(t_0)|\Delta t} e^{I_{S(t_0)}\varphi(t_0)}$$

and

$$\lim_{\Delta t \to 0} \frac{e^{I_{S(t_0+\Delta t)}\varphi(t_0+\Delta t)} - e^{I_{S(t_0)}\varphi(t_0)}}{\Delta t} =$$

$$\lim_{\Delta t \to 0} \frac{(1 - I_H(t_0)|H(t_0)|\Delta t) e^{I_{S(t_0)}\varphi(t_0)} - e^{I_{S(t_0)}\varphi(t_0)}}{\Delta t} = -I_H(t_0)|H(t_0)| e^{I_{S(t_0)}\varphi(t_0)}$$

That gives the Schrodinger equation:

$$-\frac{\partial}{\partial t} e^{I_{S(t)}\varphi(t)} = I_H(t)|H(t)| e^{I_{S(t)}\varphi(t)}$$

That means that the Schrodinger equation defines infinitesimal changes of wave functions under Clifford translations along big circles of $\mathbb{S}^3$**.**

4. Entanglement in Measurements

Whilst the Schrodinger equation governs infinitesimal transformations of a wave function by Clifford translations a finite Clifford translation moves a wave function along a big circle of $\mathbb{S}^3$ **by any Clifford parameter.**

In G_3^+ multiplication is:

$$g_1 g_2 = (\alpha_1 + I_{S_1}\beta_1)(\alpha_2 + I_{S_2}\beta_2) = \alpha_1\alpha_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_{S_1}I_{S_2}\beta_1\beta_2$$

It is not commutative due to the not commutative product of bivectors $I_{S_1}I_{S_2}$. Indeed, taking vectors to which I_{S_1} and I_{S_2} are dual: $s_1 = -I_3 I_{S_1}$, $s_2 = -I_3 I_{S_2}$, we have, see [2], sec.1.1:

$$I_{S_1}I_{S_2} = -s_1 \cdot s_2 - I_3(s_1 \times s_2)$$

Then:

$$g_1 g_2 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3(s_1 \times s_2)\beta_1\beta_2$$

and

$$g_2 g_1 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_3(s_1 \times s_2)\beta_1\beta_2$$

I the case when both elements are of exponent form:

$$e^{I_{S_1}\varphi_1} = \alpha_1 + I_{S_1}\beta_1 = \alpha_1 + \beta_1 b_1^1 B_1 + \beta_1 b_1^2 B_2 + \beta_1 b_1^3 B_3$$
$$e^{I_{S_2}\varphi_2} = \alpha_2 + I_{S_2}\beta_2 = \alpha_2 + \beta_2 b_2^1 B_1 + \beta_2 b_2^2 B_2 + \beta_2 b_2^3 B_3,$$

with

$$(\alpha_1)^2 + (\beta_1)^2((b_1^1)^2 + (b_1^2)^2 + (b_1^3)^2) = (\alpha_1)^2 + (\beta_1)^2 = 1$$
$$(\alpha_2)^2 + (\beta_2)^2((b_2^1)^2 + (b_2^2)^2 + (b_2^3)^2) = (\alpha_2)^2 + (\beta_2)^2 = 1,$$

as in the case a wave function and Clifford translation, we get:

$$e^{I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} = \cos\varphi_1\cos\varphi_2 + (s_1 \cdot s_2)\sin\varphi_1\sin\varphi_2 + I_3 s_2\cos\varphi_1\sin\varphi_2 + I_3 s_1\cos\varphi_2\sin\varphi_1 - I_3(s_2 \times s_1)\sin\varphi_1\sin\varphi_2$$

Then it follows that two wave functions are, in any case, connected by the Clifford translation (Note 3):

$$e^{I_{S_2}\varphi_2} = (e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1}) e^{I_{S_1}\varphi_1} \equiv Cl(S_2, \varphi_2, S_1, \varphi_1) e^{I_{S_1}\varphi_1},$$

Where $Cl(S_2, \varphi_2, S_1, \varphi_1) \equiv e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1} = \cos\varphi_1\cos\varphi_2 + (s_1 \cdot s_2)\sin\varphi_1\sin\varphi_2 + I_3 s_2\cos\varphi_1\sin\varphi_2 + I_3 s_1\cos\varphi_2\sin\varphi_1 + I_3(s_2 \times s_1)\sin\varphi_1\sin\varphi_2$.

This result of Clifford translation is a G_3^+ element. From knowing Clifford translation connecting any two wave functions as points on $\mathbb{S}^3$ *it follows that the* result of measurement of any observable C by wave function $e^{I_{S_1}\varphi_1}$, for example $e^{-I_{S_1}\varphi_1} C\, e^{I_{S_1}\varphi_1} \equiv C(S_1, \varphi_1)$, immediately gives the result of (not made) measurement by $e^{I_{S_2}\varphi_2}$:

$$e^{-I_{S_2}\varphi_2} C\, e^{I_{S_2}\varphi_2} = e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1} C e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1}\, e^{I_{S_2}\varphi_2} = e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1}\, C(S_1, \varphi_1) e^{-I_{S_1}\varphi_1} e^{I_{S_2}\varphi_2}$$
$$= Cl(S_2, -\varphi_2, S_1, -\varphi_1) C(S_1, \varphi_1) \overline{Cl(S_2, -\varphi_2, S_1, -\varphi_1)}$$

When assuming observables are also identified by points on $\mathbb{S}^3$ *and thus are connected by formulas as the above one we get that measurements of any amount of observables by arbitrary set of wave functions are simultaneously available.*

This is geometrically clear and unambiguous explanation of strict connectivity of the results of measurements instead of quite absurd “entanglement” in conventional quantum mechanics.

5. Conclusions

The suggested formalism gives different, more physically feasible, not mysterious, explanation of what the entanglement. It demonstrates that the core of future quantum computing should not be in entanglement which only formally follows in conventional quantum mechanics from representation of the many particle states as tensor products of individual states. The core of quantum computing scheme should be in manipulation and transferring of wave functions on $\mathbb{S}^3$ as operators acting on observables and formulated in terms of geometrical algebra. In this way quantum computer will be a kind of analog computer keeping and processing information by sets of objects possessing infinite number of degrees of freedom, contrary to the two value bits or two-dimensional Hilbert space elements, qubits.

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Notes

Note 1. In the current formalism scalars can only be real numbers. “Complex” scalars make no sense anymore.

Note 2. This is the G_3 form of a Hamiltonian in one-to-one map with its matrix form in the Pauli matrix basis, see [5]

Note 3. It is universally possible due to the hedgehog theorem.