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Integrated Teaching Design of Junior High School "Parallelograms" Unit Based on UbD Theory

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Abstract

The "Mathematics Curriculum Standards for Compulsory Education (2022 Edition)" require "teachers to conduct overall teaching design for each unit to promote students' understanding of knowledge". The teaching design using UbD method, through three stages of "determining expected results", "seeking evaluation evidence", and "designing learning activities", can enable students to achieve true understanding and overcome the drawbacks of traditional education such as "indoctrination teaching" and "isolated teaching activities". Firstly, propose the design basis for the overall unit teaching design based on UbD theory; Secondly, conduct a textbook analysis of the "parallelogram" unit in middle school mathematics published by People's Education Press in terms of "textbook content" and "core content"; Again, based on the UbD theory, a unit based overall teaching design for "parallelogram" is carried out, and the first lesson of the first section, "Properties of Parallelograms", is taken as an example for lesson teaching design; Finally, reflect on the teaching design of the "parallelogram" unit based on UbD theory.

Keywords: UbD, unit, parallelogram

1. Ask Questions

With the deepening of basic education curriculum reform, teaching concepts are shifting towards a greater emphasis on the intrinsic relevance of knowledge and the role of unit based teaching The "Mathematics Curriculum Standards for Compulsory Education (2022 Edition)" (hereinafter referred to as the "Standards 2022") require the promotion of overall teaching design for each unit, the determination of unit teaching objectives, the integration of teaching content, and the gradual implementation in each class hour to promote students' overall understanding and grasp of mathematics teaching content, and gradually cultivate students' core literacy In traditional teaching, teachers' blindly imparting knowledge and isolated teaching activities cannot make students truly understand knowledge, and teachers cannot judge whether students truly understand knowledge. Understanding by Design (UbD), also known as reverse instructional design, can address the drawbacks of traditional teaching methods that rely on rote learning and isolate teaching activities. The unit teaching design based on UbD theory emphasizes the core concepts of students' true understanding of the essence of the subject, and enables teachers to establish evaluation evidence for students' understanding, thereby truly implementing core competencies into the classroom.

2. Design Basis

This article is based on the six core concepts of UbD theory, namely "understanding six aspects", "big concepts", "basic problems", "WHERETO elements", "expressive tasks", and "evaluation and feedback". Their connotations are shown in Table 1.

3. Results

In the Results section, summarize the collected data and the analysis performed on those data relevant to the discourse that is to follow. Report the data in sufficient detail to justify your conclusions. Mention all relevant results, including those that run counter to expectation; be sure to include small effect sizes (or statistically nonsignificant findings) when theory predicts large (or statistically significant) ones. Do not hide uncomfortable results by omission. Do not include individual scores or raw data with the exception, for example, of single-case designs or illustrative examples. In the spirit of data sharing (encouraged by APA and other professional associations and sometimes required by funding agencies), raw data, including study characteristics and indivldual effect sizes used in a meta -analysis, can be made available on supplemental online archives.

Table 1. The Six Core Concepts and Connotation of UbD Theory

Name	Implication
Understanding the Six Sides	Students' understanding of knowledge needs to be achieved in six aspects: "explanation", "clarification", "application", "insight", "immersion", and "self-awareness", which also serve as important indicators for teachers to determine evaluation evidence.
grand concept	The application of disciplinary 'big concepts' can connect various scattered knowledge points and effectively integrate teaching knowledge and skills. In UbD based instructional design, big concepts run through the entire process.
fundamental problems	The fundamental question is an inspiring question that can promote exploration, understanding, and learning transfer. It can enable students to engage in necessary and continuous reflection on major concepts and previously learned knowledge
WHERETO element	WHERETO elements are involved in the development of learning activities. Ge Mengyuan et al. summarized the elements of WHERETO
Performance tasks	Present real problems to learners, similar to adults applying the knowledge from this unit in practice, to help students achieve a true understanding of the knowledge.
assessing	The "Teaching Design for Pursuing Understanding" proposes a continuous system of evaluation from simple to complex, from short-term to long-term, from non real situations to real situations, and from highly structured to unstructured, and proposes different levels of evaluation methods for different types of knowledge content.

3.1 Textbook Content

The textbook is selected from Chapter 18 "Parallelograms" of the People's Education Press Junior High School Mathematics, “Parallelogram” serves as the overarching concept and encompasses its properties, conditions for determination, and related theorems. Meanwhile, “special parallelograms” are further categorized into rectangles, rhombuses, and squares, each covering their respective properties, determination criteria, and relevant theorems—together forming a well-structured knowledge system.

3.2 Grand Concept

A mathematical concept may involve some of the five perspectives: "core concepts," "thinking methods," "important skills," "general problem-solving approaches," and "mathematical concepts After analyzing the content of the textbook, the mathematical concepts of this chapter are summarized as: reduction, deduction, abstraction ability, reasoning ability, and quadrilateral.

3.3 Main Line

This chapter mainly studies parallelograms in polygons and special parallelograms, and proves the median line theorem of triangles and the hypotenuse midline theorem of right angled triangles through knowledge of parallelograms and rectangles, respectively. In solving complex graphics, it is necessary to find the parallelogram and special parallelogram in the complex graphics based on the known corner relationships given in the problem, and use the properties and judgment theorems learned in this unit to solve them.

4. Unit Integrated Teaching Design

4.1 Stage 1: Determine Expected Teaching Objectives

Firstly, it is necessary to determine the unit teaching objectives and basic questions, estimate the level of understanding that students can achieve, and the knowledge and skills that students can master, as shown in Table 2.

Table 2. Expected Teaching Objectives and Results of "Parallelograms"

Stage 1- Expected Results
Identified goals: 1. Master the geometric features of parallelograms, rectangles, diamonds, and squares. 2. Using the properties and judgments of four types of graphics to solve complex graphic problems in practice. 3. Improve students' abstract and reasoning abilities by solving proof questions. 4. Use charts or mind maps to represent the relationships between parallelograms, diamonds, rectangles, and squares.

5. Prove the principle of equal distance between parallel lines and the median line theorem of triangles through the properties and judgments of parallelograms; Prove the theorem of the hypotenuse and midline of a right angled triangle through the properties of rectangles. 6. Use deductive reasoning. The process of deriving the judgment theorem for a parallelogram by referring to the property theorem of a parallelogram, and obtaining the judgment theorem for a special parallelogram through the property theorem of a special parallelogram. 7. When using the normalization method to solve unknown graphical proof problems, auxiliary lines are used to transform them into problems that utilize the properties of learned graphics and judgment theorems for proof.	
Expected understanding: 1. By constructing the median lines of parallelograms, special parallelograms, and triangles, complex corner relationships in graphics can be demonstrated. 2. Parallelograms, rectangles, diamonds, and squares are closely related. 3. By changing some of the corner relationships, it is possible to convert parallelograms, rectangles, diamonds, and squares. 4. Diagonals play an important role in proving property theorems and judgment theorems.	**Basic question:** 1. Why do we need to learn about parallelograms, special parallelograms, and the median line theorem of triangles? 2. What is the connection between parallelograms, diamonds, rectangles, and squares? 3. Why do we associate parallelograms, diamonds, rectangles, and squares? 4. Why do we need to learn the diagonals of parallelograms, diamonds, rectangles, and squares?
Students will know: 1. The application of parallelograms, rectangles, diamonds, and squares in daily life. 2. The property theorem, judgment theorem, and symmetry of four types of shapes. 3. The median line theorem of triangles and the hypotenuse midline theorem of right angled triangles, as well as their proofs. 4. The conditions for the mutual transformation of four types of graphics. 5. The similarities and differences in the process of proving the property theorems and judgment theorems of four types of graphs. 6. Diagonals and auxiliary lines play a significant role in graphic proof problems.	**Students will be able to:** 1. Using auxiliary lines and diagonals to prove the property theorems and judgment theorems of parallelograms and special parallelograms, as well as the median line theorem of triangles. 2. Describe orally the connections between the four types of graphics. 3. Using parallelograms to prove that the median line theorem of a triangle is equal to the distance between parallel lines. 4. Using rectangles to prove the theorem of the hypotenuse and midline of a right angled triangle. 5. Determine the corner relationships of complex shapes containing parallelograms, rectangles, diamonds, and squares based on known conditions.

4.2 Stage 2: Finding Suitable Evaluation Evidence

After determining the unit teaching objectives and estimating student learning outcomes, performance tasks are needed as evidence to evaluate whether students have achieved true understanding. In addition, more evidence is needed to prove that students have achieved the expected results in stage 1, and students are required to conduct self-assessment and feedback, as shown in Table 3.

Table 3. Evidence for Teaching Evaluation of "Parallelograms" Unit

Stage 2- Evaluating Evidence
Performance tasks: 1. Write a math essay that presents pictures of actual objects in daily life that contain rectangles, diamonds, and squares. Use tools such as rulers and protractors to prove in various ways that they are the special right angled triangles you want to showcase. 2. There is at least one example problem under each knowledge point in the textbook. Please create a new question based on the knowledge point and write the answer on another piece of paper. Then, give the question to your desk mate to do. After your desk mate finishes it, you will correct it. If both your and your desk mate's answers are correct, compare your and your desk mate's methods to see which one is easier. 3. Xiaoming wants to learn about trapezoids. Assuming you are the textbook author, please write a "trapezoid" based on the "parallelogram" in this chapter. Requirements: 1. It involves the definition of trapezoids and special trapezoids, including right angled trapezoids and isosceles trapezoids; 2. Attempt to use trapezoids and special

trapezoids to prove the median line theorem of triangles, the "equal distance between parallel lines" theorem, and the hypotenuse midline theorem of right angled triangles. If unable to prove, explain the reason to Xiao Ming. 4. Organize examples and exercises in textbooks that use auxiliary lines and diagonals to solve problems, explain the role of auxiliary lines and diagonals in solving problems, and find other ways to use auxiliary lines.
What additional evidence needs to be collected based on the expected results of Phase 1?
Other evidence: 1. Classroom observation and questioning: Check students' understanding and memory of basic knowledge, as well as observe students' participation in answering questions. 2. Homework: Check students' understanding and review of knowledge. 3. Paper and pencil test: Set proof questions to check if students can solve problems with appropriate theorems. 4. Self evaluation form: After completing this chapter, distribute a self-evaluation form to students to record their understanding of each theorem.
Student self-assessment and feedback: 1. Conduct self-assessment based on problem-solving time, accuracy, and the ability to redo mistakes. 2. Know in group discussions whether your method is the simplest and your shortcomings. 3. Can an appropriate theorem be used to solve problems during paper and pencil testing, and what is the reaction time for this theorem? 4. Write an article discussing your strengths and weaknesses in studying this chapter.

4.3 Stage 3: Design Learning Activities

In Stage 3, it is necessary to develop learning activities around the overarching concept, based on the unit teaching objectives and basic questions of Stage 1, the expressive tasks and WHERTO elements of Stage 2.

Table 4. "Parallelograms" Unit Teaching and Learning Activity Plan

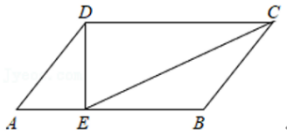
Stage 3- Design Learning Activities
Learning activities: 1. Classroom question: What do you think is the difference between a rectangle and a square? W, H The teacher introduces the basic questions, learning objectives, performance tasks, and lesson schedule of this unit. W 2. Puzzle game: Use several right angled triangles to assemble parallelograms, diamonds, rectangles, and squares and prove them together 3. The median line theorem of an equilateral triangle and the hypotenuse midline theorem of a right angled triangle. Exhibition of works that complete tasks and use the fewest number of right angled triangles. E, H 4. Each student draws Venn diagrams on their workbook to represent parallelograms, rectangles, diamonds, and squares. 5. Rationship and explain what each part represents. E, T, O 6. Teachers and students explore various proof methods for the property theorem and judgment theorem of parallelograms, and summarize the simplest method. E 7. After studying the property theorem of a certain shape, divide into groups to discuss the judgment theorem of this shape, and have the group representative give a report. E, O 8. Do it: Use tools such as pencils and rulers to indicate the distance between two parallel lines l_1 and l_2. 9. The teacher explained the distance and theorem between two parallel lines through train tracks. E, T 10. Organize case studies of using auxiliary lines to prove problem-solving in textbooks, and consider other problem-solving methods. R 11. Fill out the self-evaluation form and select the knowledge points that still have doubts. E-2, T 12. After the questions are set, the desk mates exchange questions with each other, and the question setter corrects them after completing the questions. T, O 13. Write an article summarizing the difficulties you encountered while studying this chapter and the methods you used to overcome them, and stating where you still need to work hard. T, O 14. Regarding the properties and determination steps of a special parallelogram, use parentheses to explain the property theorem or determination theorem used in this step. E2, O 15. Conduct a chapter test to combine two or more of the four types of graphics into complex graphics, set different question types and problem scenarios, and test students' ability to analyze problems and graphics. E-2

5. Lesson Teaching Design

The teaching design for each lesson should also follow the order of "determining expected teaching objectives → finding suitable evaluation evidence → designing learning activities". The following is an example of lesson teaching design for the first lesson of the first section, "The Properties of Parallelograms" (as shown in Table 5).

Table 5. "Properties of Parallelograms" Lesson Teaching Design

project	specific content
learning goal	1. By exploring the common features of the discovered instances of parallelograms, understanding the concept of parallelograms, and being able to determine whether a quadrilateral is a parallelogram. 2. Explore and prove the property theorems of parallelograms through activities such as observation, conjecture, and verification. 3. Understand the meaning of the distance between two parallel lines and be able to measure the distance between two parallel lines.
Evaluate evidence	1. Regarding performance tasks: (1) Students can analogize the concept of trapezoids from the concept of parallelograms. (2) For Example 1 and Example 2, the student can assign a "follow-up exercise" to their desk mate and correct it. (3) Students can explain to their parents why the shapes displayed in class are parallelograms, and write the explanation process into a math paper. 2 Other: (1) Classroom observation: 1. Students can prove that the two opposite sides of the displayed parallelogram are parallel by using tools such as protractors and proving that the same side inner angles complement each other and two straight lines are parallel. 2. In Activity 4, students are able to independently create an auxiliary line and prove the properties of a parallelogram through triangle congruent geometry. After joint exploration and discussion between teachers and students, students can create two types of auxiliary lines; Students can directly prove that the diagonals of a parallelogram are equal through the definition of a parallelogram. 3. Students may have a misunderstanding in Activity Seven that the line segment representing the distance between two straight lines is AD or BC, but after Activity Eight, they are able to correctly make the line segment representing the distance between two parallel lines. (2) Classroom Questions: 1. When students answer the basic questions in Activity 2, they can involve "parallel lines", "triangle congruent", "reasoning", and "practical applications of parallel lines". When answering the questions in Activity 3, be able to give the definition of trapezoid by analogy with the definition of parallelogram. 2. Students can state the important role of auxiliary lines in proving the properties of parallelograms, and know that proving "diagonals of parallelograms are equal" directly is easier than proving auxiliary lines. 3. Students can deduce from the properties of parallelograms that two lines can draw an infinite number of parallel lines between them, and any two parallel line segments between two parallel lines are equal. (3) Capable of independently completing the tasks in the "Little Trial Bull Knife". (4) Be able to write the proof steps for Example 1 and Example 2 in a standardized manner, and be able to use the method of using auxiliary lines to solve the problem in "Small Test Bull Knife 4".
learning activities	1. Activity 1: Each student should find a picture of a parallelogram in their daily life to prove that it is the parallelogram they are looking for and whether the shape they are looking for is axisymmetric. 2. Activity 2: Teacher's basic question: Why should we learn parallelograms? Answered by students. 3. Activity 3: Students use the concept of parallelograms as an analogy to give the concept of trapezoids.

	4. Small trial cow knife 1: In the quadrilateral PQMN, it is known that PQ ∥ MN, PQ=MN. Draw the quadrilateral PQMN and prove that it is a parallelogram. 5. Activity 4: Students speculate on the quantity relationship of the corners of the displayed parallelogram, and teachers and students jointly explore the proof method of the properties of the parallelogram, searching for the simplest proof method. Finally, summarize the first two properties of parallelograms. 6. Trial Bull Knife 2: Doing Example 1 in the textbook. 7. Activity 5: Summarize the two methods of using auxiliary lines in Activity 1, and the teacher will introduce the diagonal lines of a parallelogram. Draw two diagonal lines AC and BD in the same parallelogram, and students will guess the quantitative relationship between OA, OB, OC, and OD and prove it. Finally, they will obtain the third property of the parallelogram. 8. Trial Bull Knife 3: Doing Example 2 in the textbook. 9. Trial Bull Knife 4: As shown in the figure, in ▱ ABCD, CE divides ∠ BCD evenly, intersects AB at point E, EA=3, EB=5, ED=4. Find the length of CE.  10. Activity 6: For Example 1 and Example 2, give your desk mate a "follow-up exercise" that requires consistency with the knowledge points tested in Example 1 and Example 2. After the questions are given, the desk mate will exchange them and the question setter will correct them. If both answers are correct but the methods are different, it will be easier to discuss whose method is used. 11. Activity 7: Encode the vertices of the displayed parallelogram clockwise as A、 B、C、D，think about the distance between AB and CD and draw the line segment represented by the distance with a pencil and ruler, then have a group discussion. 12. Activity 8: Students do it hands-on: How many sets of parallel line segments can be drawn between two parallel lines? What is their quantitative relationship?
school assignment	1.Explain to your parents why the shape you presented in class is a parallelogram, and write your explanation process into a math paper. 2. The process of using auxiliary lines to prove the properties of parallelograms and the small trial knife will be organized in the "Auxiliary Lines" area of the exercise book, explaining the role of auxiliary lines in the proof process. 3. In addition to the example of sleepers introduced by the teacher, students are looking for another example of using the distance between parallel lines in daily life.

6. Reflection on Unit Teaching Design

6.1 The Overarching Concept Serves as a Clue Throughout the Entire Teaching Process

The big concept directly participates in the determination of learning objectives and basic problems in Stage 1. In Stage 2, expressive tasks are developed around the improvement of students' abstract and reasoning abilities and the flexible application of deductive methods. In Stage 3, learning activities are determined based on the predetermined learning objectives in Stage 1. However, due to the indirect impact of the big concept on stage 3, actual teaching may deviate from the big concept and cannot connect fragmented knowledge. Therefore, it is important to focus on teaching around the big concept.in Table 5).

6.2 Promote the Transfer from Parallelograms to Triangles and Special Parallelograms Through Expressive Tasks

The ability to complete performance tasks is the ultimate evidence for evaluating whether students have achieved true understanding. Being able to complete performance tasks indicates that students can proficiently apply the knowledge they have learned in a larger environment outside of school, and they will achieve true understanding. Both stage 3 learning activities and actual teaching activities revolve around expressive tasks, so teachers should not only focus on students' knowledge mastery in each stage, but also continuously improve their ability to transfer and apply knowledge, in preparation for the completion of expressive tasks.

6.3 Value Student Peer Evaluation and Self-Evaluation

In UBD based classroom teaching activities, "students ask each other questions and correct them" and "student self-evaluation forms" are important methods for evaluating students' understanding of knowledge. The task of "students answering questions and correcting each other" is an important expressive task. Whether the students who set the questions have the ability to do so, whether the students who solve them can solve the problems, whether the questioner can point out the mistakes in the questioner's answer, and whether the questioner can judge which one is simpler between the standard answer and the questioner's answer are all important criteria for evaluating whether the teaching objectives have been achieved and whether the students have truly understood; Through students' self-evaluation forms, teachers can identify their weaknesses, facilitate individualized teaching, and provide important references for the next stage of teaching.

6.4 Enhance One's Own Abilities and Engage in Mathematical Activities that Enable Students to Truly Understand Knowledge

Teachers should improve their ability to design teaching activities and avoid the drawbacks of "isolated teaching activities" in traditional teaching. When designing unit teaching, teachers should summarize the expressive tasks based on the overall objectives of the unit, and then divide the expressive tasks into small tasks and assign them to each lesson to achieve the lesson objectives. So teachers need to constantly accumulate teaching experience, improve their understanding of textbooks and their own teaching abilities, in order to develop excellent performance tasks and carry out meaningful teaching activities.

References

Ge, L., & Shi, M. (2020). Unit teaching design based on UbD theory—Taking plane analytic geometry as an example. *Journal of Mathematics Education*, (5), 25–31.

The Ministry of Education of the People's Republic of China. (2022). *Mathematics curriculum standards for compulsory education (2022 edition)*. Beijing: Beijing Normal University Press.

Wang, H., & Wu, Y. (2022). Exploration and practice of integrated teaching based on mathematical units: A problem-driven perspective. *Mathematics Bulletin*, (3), 27–32, 46.

Wang, X. (2018). The connotation, significance, and acquisition methods of the "big concept" of disciplines. *Teaching and Management*, (8), 86–88.

Wiggins, G., & McTighe, J. (2017). *Teaching design for pursuing understanding* (Version 2, Y. Hanbing, S. Xuelian, & L. Ping, Trans.). Shanghai: East China Normal University Press.

Xia, F. (2022). Extraction and argumentation of "mathematical concepts." *Basic Education Curriculum*, (101), 87–94.

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