International Educational Research; Vol. 8, No. 4; 2025
ISSN 2576-3059 E-ISSN 2576-3067
https://doi.org/10.30560/ier.v8n4p1

Research on the Teaching Design of Deep Learning in High School Mathematics Based on Problem Solving — Taking the Law of Total Probability as an Example

Chang Jin[1]

[1] Henan Normal University, China

Correspondence: Chang Jin, Henan Normal University, Xinxiang, Henan, 453000, China.

Received: June 22, 2025; Accepted: July 6, 2025; Published: July 7, 2025

Abstract

With the rapid development of information technology and artificial intelligence, deep learning has gradually become a major research focus in the field of education. This paper designs an innovative teaching plan for high school mathematics based on problem-solving, using the law of total probability as an example. It is widely agreed that deep learning requires students to face challenging problems and understand the knowledge in the process of solving them. Studies show that problem-oriented teaching strategies can stimulate students' mathematical thinking and promote deep understanding of concepts and formulas. Through case analysis, this research proposes an effective classroom approach for deep learning in teaching the law of total probability, aiming to improve students' mathematical competency.

Keywords: problem solving, deep learning, law of total probability, teaching design, high school mathematics

1. Introduction

Deep learning, as an advanced learning method, has been widely applied in the field of artificial intelligence. Its concepts and methods can also positively impact the traditional education system. Mathematics, as a highly logical and abstract discipline, faces the challenge of promoting students' deep understanding of concepts and their flexible application during the teaching process. The law of total probability is one of the key topics in high school mathematics, specifically in probability theory and mathematical statistics, and serves as a fundamental tool for understanding probability theory. In teaching practice, how to help students overcome the difficulty of abstract concepts and develop their ability to apply probability formulas to solve real-world problems has become an important issue for educators. This paper aims to design a teaching plan for the law of total probability based on deep learning through the "problem-solving" teaching strategy, facilitating the transition of students from learning knowledge to applying it and exploring how deep learning can enhance students' mathematical abilities.

2. Theoretical Foundations of Problem-Solving in High School Mathematics Deep Learning

2.1 Problem-Solving Teaching Model

The problem-solving-oriented teaching model emphasizes that students, when faced with complex problems, actively think, discuss, and experiment to independently or collaboratively solve problems, thereby internalizing and transferring knowledge. Unlike traditional knowledge transmission methods, the process of scientific inquiry is not only the process of solving scientific problems but also an effective material for implementing deep learning. The problem-solving model focuses more on students' hands-on practice and exploration.

In mathematics teaching, the problem-solving method effectively enhances students' mathematical abilities because students are not merely passive recipients of knowledge. Instead, they actively construct their own knowledge systems by engaging in discovering and solving problems.

2.2 Deep Learning Theory

Deep learning, originally a machine learning technique in artificial intelligence, shares a similar concept with the educational understanding of "deep learning," which emphasizes multi-level, in-depth learning that allows students to gain a more comprehensive and profound understanding of knowledge. In mathematics teaching, deep learning stresses the need for students to not only master basic mathematical knowledge but also apply this knowledge to solve complex mathematical problems and use it flexibly in practice.

Deep learning is not just about memorizing knowledge but also about understanding, applying, and innovating with knowledge. Through problem-solving, students can connect mathematical knowledge with real-world problems, thereby deepening and broadening their thinking.

3. The Relationship Between Problem Solving and Deep Learning

3.1 The Concept of Problem Solving

Problem-solving refers to students' ability to apply learned mathematical knowledge to reason, analyze, and explore uncertain, complex, or open-ended problems, ultimately finding suitable solutions. Problem solving is not just about arriving at the correct answer; it is more important to foster students' thinking and innovation abilities. The problem-solving process typically includes several steps: understanding the problem, devising a plan, and so on.

3.2 Characteristics of Deep Learning

Deep learning emphasizes not only surface-level knowledge memorization but also a deep understanding and flexible application of knowledge. The core characteristics of deep learning include:

Constructive Understanding: Students can actively learn and construct knowledge based on existing knowledge.

Long-Term Memory Consolidation: Students are able to store learned knowledge long-term and apply it flexibly in various contexts.

Metacognitive Ability: Students can self-monitor their learning process, adjust learning strategies, and reflect on learning outcomes.

Ability to Solve Real-World Problems: Students can apply mathematical knowledge to real-world problems, demonstrating strong creativity and critical thinking.

3.3 The Integration of Problem Solving and Deep Learning

Problem solving and deep learning complement each other. Problem solving provides students with contexts and challenges, while deep learning provides the necessary knowledge framework and thinking tools. Through problem-solving, students can deepen their understanding of mathematical concepts and cultivate their ability to analyze, reason logically, and think critically in practical applications. Therefore, integrating problem solving with deep learning will significantly promote the development of students' mathematical abilities.

4. Teaching Design for Deep Learning of the Law of Total Probability in High School Mathematics Based on Problem Solving

4.1 Setting Clear Goals and Guiding Learning

4.1.1 Combine Classical Probability Models to Understand the Process of Deriving the Law of Total Probability Using the Addition and Multiplication Rules of Probability

4.1.2 Understand the Form of the Law of Total Probability and be able to Use it to Calculate Probabilities

Focus: Understanding the form of the law of total probability and being able to apply it to calculate probabilities.

Difficulty: Understanding the form of the law of total probability and being able to apply it to calculate probabilities.

4.2 Analysis of Student Conditions and Textbook Analysis

4.2.1 Analysis of Student Conditions:

This lesson is an extension and expansion of the basic knowledge of probability that students have already learned, and it is part of Chapter 7 "Random Variables and Probability Distributions" from the 2019 edition of the People's Education Press A-level textbook, volume 3.

First, students have learned classical probability models and conditional probability and already have a basic understanding of the concepts of probability, conditional probability, and mutually exclusive events. However, the law of total probability, as an important tool in probability calculation, requires students to have strong logical thinking skills.

When learning the law of total probability, students usually have already mastered basic probability theory knowledge, such as conditional probability, the union and independence of events, etc. If students have a strong foundation and can understand and use conditional probability, the introduction of the law of total probability will be relatively smooth.

However, if students do not have a solid understanding of conditional probability, more time will need to be spent explaining the meaning of conditional probability, especially how to derive the total probability using the probabilities of different conditions. For some students, directly introducing the law of total probability may seem abstract, especially when dealing with multiple conditions and complex calculations, which could be confusing. Therefore, the teaching design should help students better understand the application of the formula through specific cases or examples from real life.

4.2.2 Textbook Analysis

This lesson is taken from Chapter 7 "Random Variables and Probability Distributions" in the 2019 edition of the People’s Education Press A-level high school mathematics textbook, which focuses on the "Law of Total Probability."

The law of total probability is an important tool in probability theory. It utilizes the mutual exclusiveness and completeness of events to calculate the probability of a complex event by summing the probabilities of the event under all possible conditions. In this section, the textbook first reviews basic probability concepts and formulas, laying the foundation for students to learn the law of total probability. Next, the textbook introduces the concept of conditional probability and explains in detail how to calculate it, which sets the stage for the derivation and application of the law of total probability.

When explaining the law of total probability, the textbook uses specific examples to show how the law can be applied to solve real-world problems, helping students understand and master this important tool. At the same time, the textbook focuses on cultivating students' logical thinking and data analysis skills by guiding them to analyze problems, build probability models, and apply the law of total probability for problem-solving, thus enhancing their problem-solving ability.

4.3 Creating Contexts and Introducing New Knowledge

Teacher: In life, we often face many choices. When you have to make a choice, do you rely on your intuition, luck, or your ability? Suppose you are now participating in a lottery, and there are three doors in front of you, as shown in Figure 1. Only one of the doors hides a luxury car, while the other two doors hide goats. After you choose one door, the host will open one of the doors with a goat behind it and ask if you want to switch doors. This is the famous Monty Hall problem. If it were you, would you choose to switch doors?

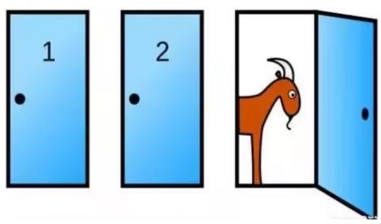

Figure 1. The Monty Hall Problem

Student 1: Switch doors.

Student 2: Do not switch.

Student 3: It depends on the probabilities.

Teacher: Exactly, whether you should switch depends on the probability of winning after switching doors. How do we calculate this? We know the probability of winning before switching doors is 1/3. What is the probability of winning after switching? With this question in mind, let’s study the law of total probability.

Teaching Intent: Start with an interesting problem to stimulate students' interest in learning and engage them actively in the classroom.

4.4 Exploring New Knowledge and Seeking Solutions

Problem Statement: We know that the order of winning and drawing is irrelevant. Next, let’s consider a few examples with common features.

Exploration: Suppose we have a bag with a red balls and b blue balls, and each time we randomly pick one ball, which is not replaced. Clearly, the probability of picking a red ball on the first draw is $P(R_1)$. What is the probability of picking a red ball on the second draw? How do we calculate this probability?

Student 1: Since the lottery is fair, the probability of picking a red ball on the second draw should be the same.

Student 2: I feel like the second draw is affected by the first draw.

Teacher: We can use the possibilities of drawing a ball to create a tree diagram, as shown in Figure 1, to solve this.

PPT display: Proof:

Let R_1 represent the event "first draw is a red ball," and B_1 represent the event "first draw is a blue ball." As shown in the diagram：Figure 2:

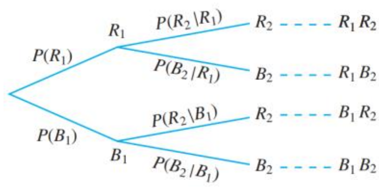

Figure 2. Tree Diagram

Teacher: From the tree diagram, we can see that there are four possible outcomes: R_1R_2, R_1B_2, B_1R_2, and B_1B_2. Now, think about it: all possible outcomes form a set, which is the entire sample space Ω. How else can we represent Ω?

Student: $\Omega = R_1 \cup B_1$

Teacher: Exactly. The event can be represented as the union of two mutually exclusive events based on the possible outcomes of the first draw:

$$P(R_2) = P(R_1R_2 \cup B_1R_2) = P(R_1R_2) + P(B_1R_2) = P(R_1)P(R_2 \mid R_1) + P(B_1)P(R_2 \mid B_1) = \frac{a}{a+b} \times \frac{a-1}{a+b-1} + \frac{b}{a+b} \times \frac{a}{a+b-1} = \frac{a}{a+b}$$

Teacher: To help you understand better, we can use a Venn diagram as shown in Figure 4. We can understand that the occurrence of R_2 is caused by either drawing a red ball or a blue ball on the first draw. This allows us to split the sample space into two parts: R_1 and B_1, and then, based on these two causes, we can find the probability of R_2 occurring. In this way, a complex event is decomposed into the sum of two mutually exclusive events, $R_2 = R_1R_2 \cup B_1R_2$.

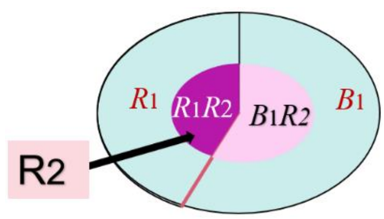

Figure 3. Venn Diagram

Summary: The method used here is to represent a complex event as the union of two mutually exclusive events and then calculate the probability using the addition and multiplication rules of probability.

Teaching Intent: While calculating the probability, let students understand the reasoning process and why it works as it does.

Teacher: If we change the conditions of the problem and the bag contains a red balls, b blue balls, and c yellow balls, how would we calculate the probability of drawing a red ball on the second draw? Let's have a group discussion.

Student: The approach is the same. $\Omega = R_1 \cup B_1 \cup Y_1$

$R_2 = R_1R_2 \cup B_1R_2 \cup Y_1R_2$

$P(R_2)=P(R_1R_2 \cup B_1R_2 \cup Y_1R_2)=P(R_1R_2)+P(B_1R_2)+P(Y_1R_2)$

and we can use the addition and multiplication rules to calculate the probability.

Student 2: We can use the Venn diagram to understand, as shown in Figure 4. There are three causes for R_2 to occur, so we split the sample space into three parts, and for each cause, we calculate R_2, breaking down a complex event into the sum of two mutually exclusive events.

Teacher: Now, if we change the conditions again, and the bag contains a red balls, b blue balls, c yellow balls, and m green balls, how would we calculate the probability of drawing a red ball on the second draw?

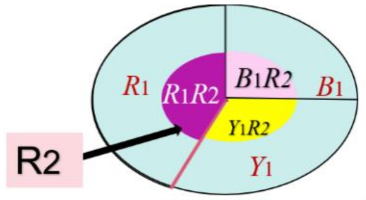

Figure 4. Venn Diagram

Student: The approach is the same.

Teaching Intent: Allow students to experience moving from specific to general cases, combining set theory, and gaining the concept and formula of the law of total probability, while also developing their core skills in logical reasoning, intuitive imagining, mathematical abstraction, and calculation.

Definition: Generally, let $A_1, A_2, ..., A_n$ be a group of mutually exclusive events, and let $P(A_1), P(A_2), ..., P(A_n)$ satisfy the condition that $\Sigma P(A_i) = 1$. Then for any event B, we have:

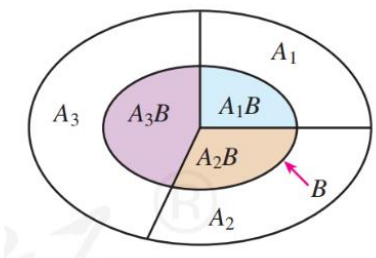

Figure 5. Venn Diagram

Figure 5. Venn diagram. The formula shown is called the law of total probability.

Teacher: The essence of the law of total probability is that the likelihood of an outcome is influenced by the "weight" of each cause.

Conditions for using the law of total probability:

①$A_1, A_2, ..., A_n$ are mutually exclusive events.

②$A_1, A_2, ..., A_n$ constitute the entire sample space.

③$P(A_1) + P(A_2) + ... + P(A_n) = 1$

④The calculation process uses the addition and multiplication rules of probability to solve it.

Teaching Intent: The teacher encourages group discussions to foster students' independent inquiry, "using mathematical insights to discover problems," and incorporates real-life examples to stimulate further exploration, deepening the classroom inquiry.

4.5 Understanding New Knowledge and Applying New Knowledge

Example: A school has two restaurants, A and B. On the first day, Wang randomly chooses a restaurant for lunch. If Wang chooses restaurant A on the first day, the probability of choosing restaurant A again on the second day is 0.6. If Wang chooses restaurant B on the first day, the probability of choosing restaurant A on the second day is 0.8. Calculate the probability that Wang will choose restaurant A on the second day.

Teacher and students analyze together: The probability of choosing a restaurant on the second day depends on which restaurant was chosen on the first day. We can represent the sample space as the union of two mutually exclusive events: "going to restaurant A on the first day" and "going to restaurant B on the first day," and use the law of total probability to solve the problem.

Let A_1 be the event "choose restaurant A on the first day," B_1 be the event "choose restaurant B on the first day," and A_2 be the event "choose restaurant A on the second day." These events are mutually exclusive. Based on the problem statement, we have:

$P(A_1)=P(B_1)=0.5$, $P(A_2|A_1)=0.6$, $P(A_2|B_1)=0.8$

Substituting the known values:

$P(A_2)=P(A_1)P(A_2|A_1)+P(B_1)P(A_2|B_1)=0.5*0.6+0.5*0.8=0.7$

So, the probability that Wang will choose restaurant A on the second day is 0.7.

Teacher: Can you summarize the approach to solving this kind of problem?

Teacher and students discuss together: Steps to calculate probability using the law of total probability:

Set the events: Treat event B as the result of a certain process, and consider A_1, A_2, …, A_n as the possible causes.

Find the probabilities: Based on known information, write down the probabilities.

Apply the formula: Use the law of total probability to calculate the probability of the result (P(B)).

Teaching Intent: Help students use the law of total probability to calculate probabilities, and develop their ability to analyze problems and apply learned knowledge to solve them. This process will help students transition from acquiring knowledge to developing problem-solving abilities.

The Monty Hall Problem: A contestant faces three closed doors, behind one of which is a car, and behind the other two are goats. After the contestant chooses a door, the host opens one of the remaining two doors to reveal a goat, and then asks whether the contestant wants to switch doors. As shown in Figure 5:

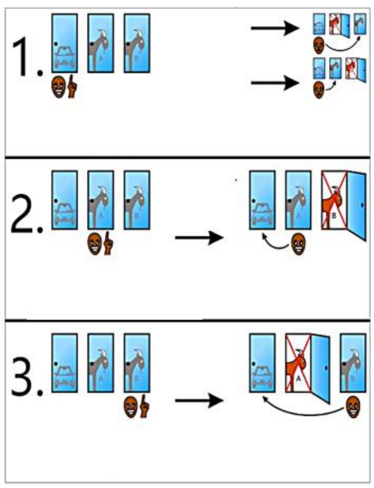

Figure 5. Switching doors result

Teacher and students discuss together: The decision to switch doors is influenced by two factors:

Student writes on the board: Solution: (1) The probability of winning the car without switching doors is 1/3, unaffected by the post-choice event.

(2) Let A_1 = "initially choosing the winning door," A_2 = "initially choosing the losing door," and B = "winning after switching." Then $\Omega = A_1 \cup A_2$, where A_1 and A_2 are mutually exclusive.

We know:

$P(A_1)=\frac{1}{3}, P(A_2)=\frac{2}{3}, P(B|A_1)=0, P(B|A_2)=1,$

$P(B)=P(A_1)P(B|A_1)+P(A_2)\cdot P(B|A_2)=\frac{1}{3}\times 0+\frac{2}{3}\times 1=\frac{2}{3}$

Teaching Intent: Traditional mathematics teaching focuses on the transmission of concepts and skills, often overlooking the value and significance of mathematics in real-world applications. However, the "learning by doing" approach emphasizes that students learn mathematics through solving real-life problems, linking mathematics to reality and developing students' problem-solving abilities and creativity.

4.6 Summarizing and Elevating Ideas

Teacher and Student Activities: Review the content learned in this lesson, following the teacher's prompts for a summary.

Mathematical Knowledge: The Law of Total Probability.

Thinking Method: From special cases to general cases.

Learning Process: The process of posing questions, discussion, experimentation, analysis, and summarizing, while incorporating core competencies.

Teaching Intent: Through summarizing, help students further consolidate the knowledge learned in this lesson and improve their ability to generalize.

Homework to Reinforce New Knowledge

Homework 1: Complete exercises from the textbook: Page 52, Exercises 1 and 2; Exercises 7.1, Questions 5, 7, and 8.

Homework 2: Corresponding supplementary materials on the "Law of Total Probability."

5. Conclusion

The problem-solving-based teaching design helps students deeply understand the Law of Total Probability and apply it to real-world problems. This teaching strategy not only enhances students' mathematical abilities but also cultivates their thinking skills and confidence in problem-solving. In future teaching, further optimization of problem design, combined with more real-life scenarios, can expand the application of the deep learning concept.

References

Wu, Y., & Wu, H. (2024). Teaching design of "gene expression" deep learning based on problem solving. *Biology Teaching*, *49*(12), 43–47.

Wang, J. (2023). Primary school mathematics teaching strategies under the "learning by use" concept — A review of *Learning by Use: Practical Teaching in Primary School Mathematics from the Deep Learning Perspective*. *Chinese Journal of Education*, *43*(10), 115.

Zhao, S., & Liu, Z. (2023). Teaching practice research on promoting deep learning under the "problem-oriented" approach — Taking primary school mathematics as an example. *Curriculum, Teaching Materials, Teaching Method*, *43*(01), 131–137.

Copyrights